Students' Views of the Relationship Between Integration and Volume When Solving Second-Semester Calculus Volume Problems

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Volume problems are a typical first type of integral application problem that students encounter in second-semester calculus. We will present students' responses to the question, "Why does integration give a volume?" Participants were Calculus 2 and Calculus 3 students enrolled in summer classes at a large, public university. Task-based interviews consisted of students working on and discussing second-semester volume problems. Students had varied and interesting responses that included formula-, units-, and derivative/antiderivative-based reasoning. These results are part of a larger study on how students understand the underlying structure of the definite integral, and how they use pictures and visualizations when solving volume problems.

Key words: Calculus, Definite Integral, Volume, Student Interviews

Second-semester calculus volume problems are a standard first step in the study of applications of integration. Previous research has found that when solving definite integral application problems, students often rely on formulas, patterns, and previously encountered methods for setting up integrals (Yeatts & Hundhausen, 1992; Grundmeier, Hansen, & Sousa, 2006; Huang, 2010). Other studies have shown that students have very little idea of the dissecting, summing, and limiting processes involved in integration (Orton, 1983; Sealey, 2006, 2014; Jones, 2015). The overarching goal of this research is to investigate how students understand the underlying sum-of-products structure of integration when solving volume problems. The focus of this poster will be on student responses to the question, "Why does integration give a volume?"

This research is built on the foundation of the constructivist learning theory (Piaget, 1970), and the framework guiding analysis of student understanding of definite integral concepts is based on Sealey's (2014) Riemann Integral Framework.

Clinical interviews were conducted with 10 students who were enrolled in Calculus 2 or Calculus 3 during the summer 2018 semester. The video-taped, one-on-one interviews involved the participants working through three volume problems and talking aloud about their thought processes and problem-solving strategies. The interviewer asked several questions throughout the interview in order to determine if the students could unpack their methods to explain why they worked. The videos are transcribed and data analysis is ongoing.

Responses to the question, "Why does integration give a volume?" varied from formula-based explanations ("Because the formula does it?") to focusing on units ("…because meter times meter times meter gives you meters-cubed which is a volume") to a derivative/antiderivative connection between volume and area ("Integrating area gives you volume"). As we continue to analyze this data, we will investigate the connection between students' responses to this question and their abilities to solve more complex volume problems.

With this research, we hope to develop activities for second-semester calculus students that emphasize their understanding of the underlying Riemann sum structure of integration and foster a deeper appreciation for how integration can be used in many different situations.

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