Graphical Forms: The Adaptation of Sherin's Symbolic Forms for the Analysis of Graphical Reasoning Across Disciplines

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This work involves a methodological presentation of an analytic framework for characterizing mathematical reasoning, introducing the construct of "graphical forms". Graphical forms build on Sherin's (2001) symbolic forms (i.e., intuitive ideas about equations) by focusing on ideas associated with a pattern in a graph. In addition to providing an overview of the symbolic forms identified in the literature, we describe how we expand the symbolic forms framework to encompass graphical reasoning. Analysis involving the graphical forms framework is illustrated by providing examples of interpretations of graphs across disciplines, using introductory biology, calculus, chemistry, and physics textbooks. Our work suggests the broad applicability of the framework for analyzing graphical reasoning across different contexts.

Keywords: Mathematical Reasoning, Symbolic Forms, Interdisciplinary

The framework we describe in this work is informed by the resource-based model of cognition, which posits that knowledge is composed of fine-grained cognitive units ("resources") that form a network and are activated in response to specific contexts (Hammer et al., 2005). Resources reflect ideas that may be characterized as conceptual, epistemological, or procedural (Becker, Rupp, & Brandriet, 2017). Symbolic forms can be considered "mathematical resources" that describe intuitive ideas associated with patterns in an equation, such as attributing the idea of "balancing" to the pattern "☐ = ☐" (a box represents a term or group of terms) (Sherin, 2001). Sherin's (2001) initial work involved characterizing algebraic operations in physics problem solving, but recent work has utilized the symbolic forms across disciplines, characterizing advanced mathematical reasoning about topics such as differentiation, integration, and vectors (Dreyfus et al., 2017; Dorko and Speer, 2015; Hu and Rubello, 2013; Izsak, 2004; Jones, 2013, 2015a, 2015b; Rodriguez et al., 2018; Schermerhorn and Thompson, 2016; Von Korrff and Rubello, 2014).

It is also worth noting that the symbolic forms framework has focused on students' reasoning, but we assert that experts have access to a similar set of mathematical resources about equations. Furthermore, we describe an analogous type of reasoning about graphs, graphical forms. Using examples from graphs presented in introductory biology, calculus, chemistry, and physics textbooks, we illustrate examples of graphical forms, including "steepness as rate" (the relative steepness of a graph provides information about rate), "straight means constant" (a straight line indicates a lack of change), and "curve means change" (a curve indicates change). Graphical forms, like symbolic forms, have broad utility and applicability for interpreting mathematical reasoning because they are not context-specific. In addition, these mathematical ideas serve as an anchor to attach meaning and describe phenomena. In this work we hope to draw attention to the role of intuitive mathematical ideas in interpreting graphs and provide a potential avenue for future research across disciplines.

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