Student Reasoning about Basis and Change of Basis in a Quantum Mechanics Problem

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In this study, we explore how quantum mechanics students understand linear algebra concepts in the context of two spin-½ probability problems, the second of which required a change of basis. In particular, our research question is: what problem solving approaches do students use, what mathematical concepts are involved in that approach, and how do students reason about basis and change of basis as they engage with the problems? Data come from individual, semi-structured interviews with twelve quantum mechanics students from two different universities. Our poster will share preliminary results for all parts of the research question.

Keywords: linear algebra, change of basis, quantum physics, student reasoning, problem solving

Several studies (Adiredja & Zandieh, 2017; Hillel, 2000; Stewart & Thomas, 2010) explore student understanding of basis. For example, Adiredja and Zandieh (2017) explored students' conceptual metaphors for basis related to real-life contexts. Students' verbs related to bases generating or describing a space, and their adjectives described bases as minimal, maximal, essential, representative, different, and non-redundant. However, little is known about student understanding of change of basis, especially in a upper-division physics context.

In our study, semi-structured interviews (Bernard, 1988) were conducted with 12 quantum mechanics students at the end of the semester. Eight were from a junior-level spins-first course at a large public research university in the northwest US, and four were from a senior-level spins-first course at a medium public research university in the northeast US. Interview questions were designed to prompt student reasoning about linear algebra concepts used in quantum mechanics. For this study, we analyzed responses to: "Consider the quantum state vector $|\psi\rangle = \frac{3}{\sqrt{13}}|+\rangle + \frac{2i}{\sqrt{13}}|-\rangle$. (a) Calculate the probabilities that the spin component is up or down along the *z*-axis. (b) Calculate the probabilities that the spin component is up or down along the *y*-axis." The follow-up question of interest was: "How do you see this problem relating to basis or change of basis?"

Through a grounded analysis (Strauss & Corbin, 1998), preliminary results indicate that to complete the spin up portion of problem (b), students used two main approaches: changing $|\psi\rangle$ to be written in terms of the *y*-basis, or changing *y*-basis vectors to be written in terms of the basis the given $|\psi\rangle$ was expressed in. The linear algebra concepts involved in at least one of these approaches include linear combinations, inner product properties for orthonormal bases, squared norms of inner products, and systems of equations. We also found some nuance in the ways that the students discussed change of basis. Students used phrases in which the object of focus is either a basis or a vector as they discussed the result of changing the basis. Of the students whose object of focus was a vector, some indicated that change of basis is a way of rewriting the vector, and some indicated that change of basis is a process of transforming the vector in a way that the post-change vector becomes a different vector than the original. Students whose object of focus was a basis referred to switching or changing the basis of a vector, rather than the changing the vector itself. Additionally, we examined the context of the phrase "in a basis" as it appeared in the students' dialogue. This is a common phrase used in many different contexts, sometimes imprecisely. We noticed that students talked about any of the following as being "in a basis": a vector, a procedure, a person, or the problem setting.

References

Adiredja, A. P., & Zandieh, M. (2017). Using intuitive examples from women of color to reveal nuances about basis. In A. Weinberg, C. Rasmussen, J. Rabin, M. Wawro, and S. Brown (Eds.), *Proceedings of the 20th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 346-359). San Diego, CA.

Bernard, R. H. (1988). *Research methods in cultural anthropology*. Newbury Park, CA: Sage Publications.

Hillel, J. (2000). Modes of description and the problem of representation in linear algebra. In J. L. Dorier (Ed.), *On the Teaching of Linear Algebra* (pp. 191-207). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Stewart, S., & Thomas, M. O. J. (2010). Student learning of basis, span and linear independence in linear algebra. *International Journal of Mathematical Education in Science and Technology*, *41*(2), 173–188.

Strauss, A., & Corbin, J. (1998). Basics of qualitative research: Techniques and procedures for developing grounded theory (2nd ed.). Thousand Oaks, CA: Sage Publications.