Mathematics of Graphic Animations of Solids of Revolution

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The purpose of this study is to look at the covariational reasoning necessary to define surfaces parametrically in 3-space and the mathematics involved in writing statements in Graphing Calculator (GC). This study offers evidence on how developed ideas of covariational reasoning and parametric explanation have an impact on visualizing animations, especially in the case of solids of revolution. Additionally, the analysis of seven calculus textbooks focusing on parametrically defined relationships reveals that the use of parametric explanation is confined to representing familiar graphs parametrically and the trends of describing parametric relationships in the textbooks will be discussed as well. Through the structured way of thinking embedded in the statements in GC, students will be able to understand how the surfaces are formed as the parameters vary.

Keywords: Covariational Reasoning, Parametric Relationship, Solid of Revolution, Technology

Project DIRACC (Developing and Investigating a Rigorous Approach to Conceptual Calculus) utilizes didactic objects to support students' dynamic imagery (Thompson, 2002). In the online calculus textbook 'Newton meets technology' (Thompson & Ashbrook) developed as a part of the project DIRACC, students can use the animations in 3-space to develop productive and meaningful images of solids of revolution.

We analyzed seven textbooks focusing on parametrically defined relationships. The trends of describing parametric relationships are disclosed as follows; 1) Parametric equations are a way of defining a 'curve' in the 'xy' plane. 2) The parametric context is only for finding $\frac{dy}{dx}$ as slope of tangent line. 3) Parametric relationships are just to use the chain rule to find $\frac{dy}{dt}$ and $\frac{dx}{dt}$.

The textbook analysis led us to ponder what would be the most problematic part for students when connecting the parametric representations with the visualization of animations. The mathematical ideas of parametrically defined relationships and covariational reasoning are correlated. In other words, the development of dx and dy as variables throughout the DIRACC textbook is crucial to show the rate at which one quantity changes with respect to the rate of change of another quantity, where these two quantities are related to a common third variable t.

Covariational reasoning and parametric reasoning are implemented in the statements of Graphing Calculator (GC), so that they provide students with the ways of thinking of parameters as varying quantities within finite sized intervals. We created four animations of revolving the graph y=sin(x) around the x-axis and the y-axis, where $x \in [o,\pi]$ using two different perspectives for each revolution; varying height and varying width. All the animations have multiple variables varying simultaneously to create the image. To understand the creation of each surface, a method we found helpful is to consider a particular value of all parameters but one and focus on the changes in the remaining parameter. By focusing on the variation in each parameter individually, we are able to put the variations together more coherently.

Through beginning with this structured way of thinking, students looking at these animations will be able to understand how the surfaces are formed as the parameters vary.

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