Different Epistemological Frames Give Rise to Different Interpretations of College Algebra Lectures, Yet Pragmatic Decisions About Grades Swamp Productive Beliefs

Suzanne Kelley
Temple University

Benajmin Spiro
Temple University

Timothy Fukawa-Connelly
Temple University

In this study, we present a comparative case study of two students with different epistemological frames watching the same college algebra lectures. We show that students with different epistemological frames can evaluate the same lectures in different ways, including very different evaluations of the goals and important content. Moreover, we illustrate that even when students have seemingly productive epistemological frames might give way to pragmatic decisions about earning a good grade when presented with too much information too fast. We argue that students might have productive dispositions towards mathematics, but default to a procedural orientation, and, as a result, appear indistinguishable in a class, from those who only have a procedural view of mathematics. These results illustrate how a student's interpretation of a lecture is not inherently tied to the lecture, but rather depend on the student and her perspective on mathematics and factors in the control of the lecturer.

Keywords: College Algebra, Evaluation of lecture, Student thinking, Epistemological frames

Sitting in a lecture is one of the most common experiences that students have in a tertiary mathematics class from introductory classes through their proof-based work (e.g., Mesa, 2018; Johnson, Keller, & Fukawa-Connelly, 2018). At the same time, there is strong agreement among mathematics educators, and some mathematicians, that lecture is ineffective at helping students learn mathematics (e.g., Bressoud, 2011). While studies have investigated the results of student learning gains and attitudes following video watching of math lectures (c.f. CITES), few studies have explored the ways that students interpret and make sense of lecture. For example, Weinberg and Thomas (2018) asked 12 students to watch calculus lectures in video form and engage in reflective dialogue in real-time. They found that students attempted to self-monitor for understanding but were often doing so in ways misaligned with mathematical meaning. While Weinberg and Thomas identified some ways that students attend to particular moments within a lecture, these perspectives all require students to make identifications of queues within the lecture. More research is needed to determine what students value in mathematics lectures and why they value these. Student beliefs about mathematics and what it means to do mathematics hold promise for better understanding what they might take from a lecture. For example, significant evidence suggests that students believe that mathematics is about following rules (e.g. Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Schoenfeld, 1989), and, consequently, Schoenfeld (1989) argued that student's evaluation of teaching depended on whether the teachers clearly presented the rules and how to use them. Similarly, other researchers have argued that student's beliefs interfere with their learning (e.g., Alcock & Simpson, 2004; Bressoud, 2016; Dawkins & Weber, 2017; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Solomon, 2006). Including the claim from Weinberg and Thomas (2018) that students are not always able to monitor their own understanding. Subsequent research has repeatedly suggested that their behaviors are indicative of their beliefs (Muis, 2015). Yet, little of this work has focused on how students interpret mathematics lectures. One notable exception is the work of Krupnik, Fukawa-Connelly, and Weber (2017) who explored how two students' differing epistemological frames (e-frames) lead to different interpretations of the same real analysis lecture. We build on this work to explore the following questions:

1. What meanings of mathematics do students have?
2. How do those meanings shape how students interpret mathematics lectures?
3. What other heuristics do students use to evaluate mathematics lectures?

Like other recent studies, we explore student's evaluation of lecture via the use of video for methodological reasons.

Theoretical Framing

Following Krupnik et al. (2017), we adopt Goffman's (1997) concept of frames, further specifying that we focus on epistemological frames, based on Redish's (2003) description. For Goffman, a frame is a means for individuals to make sense of complex social spaces. As an example of a frame, we might consider the 'art museum frame' in which someone entering an art museum would expect to find art displayed on the walls of a building for perusal. At the same time, there are expectation of behavior for people entering that social situation that include things like; 'quiet discussion' and 'don't touch the art.' Violations of these heuristics are likely to lead to some sort of social sanction. Such a frame might be counter-productive at a children's museum, such as the Please Touch Museum in Philadelphia, which encourages people to touch and interact with the exhibits.

Physics educators, Redish (2004) included, refined the notion of frame for an academic setting. These epistemological frames (e-frames) guide people's expectations for pedagogical settings such as mathematics classes. They might then not develop the desired conceptual understanding. Krupnik, et al summarized e-frames as such:

> "These consist of an individual's responses to questions such as "what do I expect to learn?" and the related questions of "what counts as knowledge or an intellectual contribution in this environment?" and "by what standards will intellectual contributions be judged?" (Redish, 2003)(p. 174, 2018)

Krupnik et al. (2017) used this notion to explore how two students reacted to the same real analysis lectures. They described Alice as holding the position that one needs to define a concept in order to reason about it, and, consequently, that making claims and providing justification requires precise definitions. Relatedly, Alice believed that providing a definition was a mathematical contribution. For Alice, this meant that the idea of re-defining the rational numbers was a mathematical contribution because then she could make claims and provide justification for those claims about the rational numbers. They claimed that Brittany did not concern herself with a formal definition, and, instead believed that definitions were better when they were comprehensible and provided new insight into a concept. Because Brittany believed that she had a strong understanding of the rational numbers, she believed the re-presentation of the rationals to be relatively useless. While she might write proofs that comply with the norms of the class, she may not substantively change her conceptions, and, miss fundamental ideas in real analysis.

We might similarly reinterpret Weinberg, Wiesner, and Fukawa-Connelly's (2014) exploration of student sense-making in abstract algebra lectures. For example, Weinberg et al. (2014) showed an example where a professor drew a diagram off to the side of the main lecture notes. The stated goal of the lecture was to define the rational numbers as a set of equivalence classes. They claimed that a student, Jocelyn, used a communication-oriented frame to determine that the diagram on the side "was not the answers that he was looking for," while noting that the diagram was a diagram representing generic equivalence classes. Another interpretation is that the instructor might believe that making explicit connections between any particular example of equivalence classes and the abstract concept is a mathematical contribution and can help students build understanding of the abstract concept. In contrast, the student might believe that answering the asked question was the meaningful mathematical contribution. Our contribution is a further exploration of the relationship between student's conceptions of mathematics and their evaluation of mathematics instruction.

Methods

Participants

We solicited the participation of four students enrolled in a College Algebra class at a large, east-coast university although we only report on two here. The university requires four college-preparatory mathematics classes for admittance, meaning all of the students had passed, at the least, a precalculus class while in high school. We note that all four of the students were intending to major in some type of non-STEM education field. We do not know how this might shape their thinking about mathematics and mathematics teaching.

Data Collection

During the first interview, participants were shown two videos. The first was primarily procedural instruction of the mathematics topic while the second was a more conceptual viewpoint of the same topic. Following each video, participants were asked the same series of questions, which included the following:

1. What did you notice about the video?
2. What did you think was important to take away? Why?
3. What in the video did you notice, but not find valuable? Why?

To further probe thinking about the video content, participants were also asked about their prior knowledge of the content, as well as if they were confused by anything in the video content or presentation and if they thought the videos were similar or different in any way. Our purpose for asking these questions was to identify initial ideas about the e-frames of the participants. After the first interview, we listened to the audio of the interviews and developed initial hypotheses about the participant's e-frames which guided our selection of the video for the second interview. For the second interview we showed the students a video that contained a mixture of procedural and conceptual content and we asked the same three-question protocol as in the first. Finally, participants were asked a series of questions to elicit information about their e-frames, including:

- What does it mean to be good at math? Why?
- What do you hope to get from attending lectures in mathematics?
- What makes a good lecture? What makes a bad lecture?
- What do you think it means to understand a mathematical concept?
- What do you think makes a good mathematical explanation of a concept?

Data Analysis

The goals of our analysis were to develop a set of claims about the heuristics students evaluate instruction and ground those heuristics in their beliefs about what it means to know and do mathematics. As a result, after transcription, we followed Mason (2002) in our analysis of the student interviews and attempted to:

(i) give an account of the e-frames that each student holds,

(ii) give an account for the evaluations that each of the students gave to the respective mathematics lectures (videos).

We first coded each student's claims about what it means to know and do mathematics. While many of these were made in response to specific prompts about these ideas, students often made unprompted comments in their other responses. We identified such instances when they made explicit claims about 'math class' or 'doing math' that moved beyond the specific context being discussed. In our next round of coding, we summarized the student's comments about the different videos. We particularly attended to two types of claims, when the students gave a statement about the mathematical goals or contribution that the professor was intending to make, or, when the students made evaluative comments and comparative comments about the mathematics of the videos. We distinguished those that focus on mathematical content and those that focus on aspects of the presentation. Then, we categorized each comment as either supporting or contradicting an e-frame for each student, or, as needed developing a new hypothesis for an e-frame. We rejected any hypotheses when we did not find sufficient support for it (e.g., few supporting claims), or, we found significant inconsistent evidence. We present the data as contrasting case studies to illustrate how these students hold different e-frames and evaluated the videos in different ways but might all appear to have a procedural focus.

Data and Results

Lauren's Conception Of What It Means To Do Mathematics: Mathematics Includes Decontextualized Problems That Can be Solved Efficiently Through Memorized Equations

Lauren believes that someone who is good at mathematics, "can solve problems really quickly and everything like that but I've come to know that it really means like memorizing equations." That is, for Lauren, being proficient at mathematics means having equations memorized that she can then use to solve posed problems. She later claimed that "having those equations memorized" was a first step towards proficiency. For Lauren, the second step towards being good at mathematics requires, "knowing which equation goes with which type of problem." We interpreted this claim as meaning that being good at mathematics requires being able to select an appropriate procedure to accomplish a required task. She later specified that she felt proficient at mathematics because "I know which equations to use, like I know how to do it at this point." She repeatedly returns to the notion that "I prefer to see the equation," because she feels that following a procedure gives certitude and she would only attempt something new or different "when you're lost and don't know what to do." But, critically, "in the box thinking (procedural) is more important because math is very straight forward." When she does mathematics, she prefers to use, one, single procedure in the way that it was taught. She stated, "I feel like I always like to use the equation," which she contrasted with "outside the box" thinking. She reiterated a nearly identical claim repeatedly in the interviews, for example later claiming, "I always just think of it as, here's an equation, plug it in, and solve. I don't really think outside of the box I guess." We summarize her perspective on mathematics as believing that mathematics is best done via procedure, and, it requires both memorizing procedures and knowing which procedure to apply at a particular time.

Lauren's Evaluation Criteria: Mathematics Instruction Is About Presenting Procedures And Explaining When They Are Used.

Lauren's Heuristic 1.1: Good mathematics instruction involves clear presentations of procedures. When Lauren evaluates pedagogical presentations she uses a variety of heuristics, all of them tied to her goals for mathematical proficiency. The primary evaluative heuristic for a pedagogical presentation that Lauren uses is the clarity with which the instructor presents the steps in a procedure and when to use it. That is, she values a clearly presented procedure with examples of the process. When presenting an example of the procedure, this should also include an explanation of how each number was derived. She gave a positive evaluation of procedural videos, repeatedly noting that they are "really clear." For example, she claims "like if you were to just skip from negative twenty to positive twenty someone else might be confused by that and then how he just wrote it out and just explained how he got each product and then which lead to the answer for y like that was clear as well." In this quote, she specifically stated that the explanation of the derivation of a particular value was "clear" and she valued that he "explained how he got each product and then which lead to the answer," we interpreted all of this as her valuation of a detailed presentation of steps, including derivation of numbers.

Moreover, her only critiques of procedural videos came when she felt that the procedures or exemplification omitted details, for example, noting, "The only thing I got confused about… when he came up with the two for the vertex, I feel like he just pulled it out of thin air." The moment referenced by Lauren occurred when the lecturer in the video derived the vertex from equation y=(x-2)^2-5. He began by stating that (x-2)^2 was necessarily greater than or equal to zero. He continued by reasoning that since the vertex was a minimum it could only occur when (x-2)^2 was zero, and hence x must be two. However, his argument was constructed from conceptual mathematical reasoning rather than procedural steps that could be followed. We interpret Lauren's reaction that the value 2 was "pulled out of thin air" as part of an e-frame in which a procedureless justification was the same as no justification at all.

Lauren's Heuristic 1.2: Good mathematics instruction involves explanations of when to use procedures. Lauren also wanted to know when to use a procedure and repeatedly praised videos that made this explicit. For example, "it was a pretty good video. I guess it's like important that you would use this equation when you have a complex equation like that one where it's not so easy to find what x is and everything so it was a good video. It was really clear." Here, her evaluative focus is that the presenter specified that a particular process or equation can be used for a particular task. When evaluating a video focused on the different forms of a quadratic function she claimed, "I liked how he made the chart showing what each equation, what you can see and what you can't see, that was really nice. Because it's just good to know … which one you wanna use, or what to expect when using it." That is, she evaluated the presentation as good because it was explicit about when to use each form, again, giving rules for accomplishing a particular task. More, she specifically stated that a lecture should help a student understand, "why you use all the equations you use" where her use of 'why' means picking the right procedure to accomplish a task. The fact that these are Lauren's primary heuristics for evaluating a pedagogical presentation in mathematics is perfectly aligned with her beliefs about what it means to do mathematics; to know procedures and when to use them.

While Lauren repeatedly claimed to value conceptual explanations and used language that suggests this, such as, that she values "knowing what different equations mean regarding the shape of the function" we interpreted her claim as being able to link the graph of the function with the symbolic form, not that she can describe *why* the graph has that particular shape. More, she repeatedly demonstrated that she is content to have only memorized procedures, repeatedly making a claim like, "I memorized it" and "it just is what it is." That is, while she might use language that appears to value conceptual understanding, she appears to mean how and when to use a procedure.

Joseph's Conception Of What It Means To Do Mathematics: Mathematics Is An Exercise In Problem-solving.

Joseph considered mathematical skill to be the process of "just being able to figure out problems and stuff," where we interpret the term problems to represent a decontextualized mathematical task. He believes that mastery of mathematics includes, "Being able to be given a problem maybe that you haven't seen before, but that connects a few of the concepts you've learned and you can reason your way around the things that you've learned to figure out what you're supposed to do about that." When discussing the goal of mathematical knowledge, or the purpose of mathematical lecture, he often referred to math's future applicability during an assessment situation. On four separate occasions he cited tests as the times he would be actively using mathematics. He gave no other examples of times in which he might use math. Additionally, he claimed to identify mathematical understanding in himself when, "–I can see a problem and particularly ... I think the most understanding is when you see a problem and you know what you can do to it and how to do it." We interpret this to mean that he views mathematics as a set of problems to be identified and solved, as opposed to a set of concepts to be applied situationally.

Joseph's Evaluation Criteria 1: Mathematics Instruction Provides the Learner with Conceptual Information that Is Pragmatically Useful.

Joseph's heuristic 1.1: Good mathematics instruction includes generalizations of conceptual information. In addition to lessening the need for memorization, Joseph considered the generalizability of conceptual information to be more powerful than specific examples and evaluated instruction positively when it was included. He stated that he would prefer instruction that included a general problem over one with specific procedures because, "you can apply it to whatever example you're using." He described his preference this way:

> And it makes it easier to understand ... 'cause sometimes on past math tests, math tests I've taken in high school, sometimes there might have been a concept I didn't really understand, and then in the middle of the test because I understood multiple concepts ... or rather not concepts, more like a problem I didn't understand ... because I understood multiple concepts I could figure it out and figure out something that I had missed or that I hadn't studied, and then I'd be able to answer the question correctly because I understood what was at work behind the stuff I was supposed to be doing. Maybe if I had forgot an equation I could figure out a different equation made up of other ones that I learned.

We interpreted this to mean that Joseph values conceptual information for its general applicability for broad swaths of problem-solving situations. Joseph applied the application of general conceptual knowledge to a specific problem-solving situation during the interview. While solving a completing the square problem, he became confused by the video's final step at the same point where Lauren did, as described in her Heuristic 1.1. However, Joseph noticed that the process of completing the square had converted the parabola's equation into vertex form and was able to identify the vertex from this context rather than attempt to replicate the presenter's reasoning, in doing so, he was able to actively apply conceptual knowledge in order to mitigate procedural confusion.

Joseph's heuristic 1.2: Good mathematics instruction involves conceptual information because it reduces the need for memorization. In evaluating a conceptual video Joseph described some information as being of the type that "you wouldn't really need to know how to do as a student, but if you understand it, it makes other things a lot easier." We took this to mean that although Joseph values more conceptual instruction, he is pragmatic in his valuation. Although he expresses that conceptual information is in itself unnecessary because it is not included on class assessments, he values this information because it can be applied to problem solving in testing situations. He specifically contrasted the instructional content in the conceptual lecture with procedural information that would be "needed for the test." Joseph recognizes conceptual information as useful because "the more you actually understand the reasons behind what you're doing, the less that you have to memorize for the test." While Joseph recognized that procedural fluency is what is assessed in exams, he also valued conceptual information from the lecture because it reduced the demands for memorization. He continued by stating, "It'd also be less memorization, of just memorizing equations and signs and stuff; you don't understand why they're the way they are." Joseph acknowledged that although conceptual understanding of mathematical situations could be, "more valuable, it also takes more time and more effort to acquire." However, he justified this burden explaining, "There's more bang for your buck, I guess you could say." Joseph values instruction that includes conceptual content because it helps him to solve problems when he cannot remember memorized information.

Discussion

The purpose of this paper was to explore the relationship between what students believe to constitute mathematically valuable activity and their heuristics for evaluating mathematical pedagogical presentations. More, unlike Krupnik et al., (2017) work, these students were enrolled in the course for which they were evaluating the presentations, and, they could ostensibly derive benefit from the videos as they had not yet taken their final exam. In each case, the students' beliefs about what constitutes mathematical activity guided their evaluations of the different pedagogical presentations. We note some limitations, we only studied 2 students and a few presentations of very limited duration. It is possible that neither the students nor the videos had sufficient variation to capture enough meaningful differences. At the same time, the two students had very different beliefs about what counts as mathematics, and, as a result, gave very different evaluations of individual lectures and even different components within the lectures. While Lauren valued only the procedures, Joseph valued conceptual explanations for a number of reasons. Yet, when those conceptual explanations might prevent the student from learning the procedure, they would stop attending to the conceptual aspects. The students made a rational decision in that both students recognized that only procedural proficiency was required to be successful on mathematics exams. Thus, we note that while to an observer, it might appear that students only value the procedural aspects of a mathematics lecture this is not necessarily true. It might be a form of coping mechanism based on a rational decision-making process. More though, it means that even though students might have productive beliefs, these might not be visible to observers, instead it might appear that all students have a procedural focus. In none of the videos did the instructor attempt to explain what it means to do mathematics. As a result, there was nothing to challenge either of the students' beliefs, meaning students could only interpret the lectures through the beliefs that they already held. Perhaps by specifically teaching about meta-mathematical issues an instructor could change what students attend to and take from a mathematics lecture. Yet, as a final note, based on the very different desires of the students in terms of detail, it would be impossible to give a mathematics lecture that satisfies all students.

References

Alcock, L., & Simpson, A. (2004). Between visual reasoning and the learner's beliefs. *Educational Studies in Mathematics*, *57*, 1–32.

Bressoud, D. M. (2011). The worst way to teach. July. Retrieved from https://www.maa.org/external_archive/columns/launchings/launchings_07_11.html

Bressoud, D. (2016). What we say/What they hear II. March 1. Retrieved from http://launchings.blogspot.com/2016/03/what-we-saywhat-they-hear-ii.html.

Brousseau, G.: 1981, 'Problèmes de didactique des décimaux', Recherches en didactique des mathématiques 2(1), 37–127.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reys, R. (1980). Results of the second NAEP mathematics assessment: Secondary school. *The Mathematics Teacher*, *73*(5), 329–338. https://doi.org/http://www.jstor.org/stable/27963780

Dawkins, P. C., & Weber, K. (2017). Values and norms of proof for mathematicians and students. *Educational Studies in Mathematics*, *95*(2), 123–142. https://doi.org/10.1007/s10649-016-9740-5

Goffman, E. (1997). The Goffman reader. New York: Wiley-Blackwell.

Johnson, E., Keller, R., & Fukawa-Connelly, T. (2018). Results from a Survey of Abstract Algebra Instructors across the United States: Understanding the Choice to (Not) Lecture. International Journal of Research in Undergraduate Mathematics Education, 4(2), 254-285.

Krupnik, V., Fukawa-Connelly, T., & Weber, K. (2017). Students' epistemological frames and their interpretation of lectures in advanced mathematics. *Journal of Mathematical Behavior*, *49*(December 2017), 174–183. https://doi.org/10.1016/j.jmathb.2017.12.001

Lew, K., Fukawa-Connelly, T. P., Mejía-Ramos, J. P., & Weber, K. (2016). Lectures in advanced mathematics: Why students might not understand what the mathematics professor is trying to convey. *Journal for Research in Mathematics Education*, *47*(2), 162. https://doi.org/10.5951/jresematheduc.47.2.0162

Mason, J. (2002). Researching your own practice: The discipline of noticing. Routledge.

Mesa, V. (2017). Mathematics Education at U.S. Public Two-Year Colleges In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 949-967). Reston, VA. National Council of Teachers of Mathematics.

Muis, K. R. (2015). Personal epistemology and mathematics : A critical review and synthesis of research, *74*(3), 317–377.

Redish, E. F. (2003). A theoretical framework for physics education research: Modeling student thinking. *The Proceedings of the Enrico Fermi Summer School in Physics*, 1–50. https://doi.org/10.1119/1.1509420

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education*, *20*(4), 338–355.

Solomon, Y. (2006). Deficit or difference? the role of students' epistemologies of mathematics in their interactions with proof. *Educational Studies in Mathematics*, *61*(3), 373–393. https://doi.org/10.1007/s10649-006-6927-1

Weinberg, A., & Thomas, M. (2018). Student learning and sense-making from video lectures. *International Journal of Mathematical Education in Science and Technology*, *5211*, 1–22. https://doi.org/10.1080/0020739X.2018.1426794

Weinberg, A., Wiesner, E., & Fukawa-Connelly, T. (2014). Students' sense-making frames in mathematics lectures. *Journal of Mathematical Behavior*, *33*, 168–179. https://doi.org/10.1016/j.jmathb.2013.11.005