Inquiry Does Not Guarantee Equity

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Our field has generally reached a consensus that active learning approaches improve student success; however, there is a need to explore the ways that particular instructional approaches impact various groups of students. Here we examined the relationship between gender and student learning outcomes in one particular context – abstract algebra, taught with an Inquiry-Oriented Instructional (IOI) approach. Using hierarchical linear modeling, we analyzed content assessment data from 522 students. While the performance of IOI and non-IOI students was similar, we detected a gender performance difference (men outperforming women) in the IOI classes that was not present in the non-IOI classes. In response to these findings, we present avenues for future research on the gendered experiences of students in such classes.

Keywords: undergraduate mathematics, gender, inquiry-oriented, assessment

Broadly speaking, 'active learning' approaches to instruction in undergraduate science, technology, engineering, and mathematics (STEM) classes have been tied to improved student success and learning, with Freemen et al.'s (2014) meta-analysis of 225 studies providing compelling evidence. Additionally, a number of more isolated studies have suggested that active learning may be more equitable for students from historically marginalized groups (e.g., Laursen, Hassi, Kogan, & Weston, 2014; Eddy and Hogan, 2014). For instance, Laursen et al.'s (2014) study found that students who took lecture-based mathematics classes exhibited substantial decreases in their mathematics self-efficacy with women disproportionately underestimating their ability. In contrast, the decrease in self-efficacy was less drastic for students in Inquiry Based Learning (IBL) classes and consistent across genders – perhaps helping to "level the playing field" (p. 415) for women and men.

However, the mechanisms linking active learning approaches to more *equitable* student outcomes are not well understood and the generalizability of these findings has been questioned (e.g., Hagman, 2017). In order to understand and replicate the positive results found for general student populations (e.g., Freeman et al., 2014) and the results for particular student groups (e.g., Laursen et al., 2014), it is important to identify the critical features of active learning that are empirically and theoretically linked to improved student outcomes. Indeed, Eddy and Hogan (2014) argue that any classroom intervention will impact different groups of students in different ways, and they extend Singer and colleagues' (2012) call for identification of critical features in order to explore the ways that particular approaches impact various student sub-populations.

In light of Laursen et al.'s findings, and in accordance with Eddy and Hogan's (2014) and Singer et al.'s (2012) call to better understand the ways in which particular instructional practices may impact particular groups of students, we examined the relationship between gender and student learning outcomes in one very specific context – abstract algebra, taught with an Inquiry-Oriented Instructional (IOI) approach supported through an ongoing and substantial professional development program. Our work draws on data collected in an NSF-funded project, Teaching Inquiry-oriented Mathematics: Establishing Supports (TIMES). The TIMES project, in an effort to support instructors learning to teach in an inquiry-oriented manner, provided participants with curricular support materials, summer workshops, and weekly online workgroups as they worked to implement a set of inquiry-oriented instructional materials. Here, we restrict our analysis to those instructors implementing the *Inquiry-Oriented Abstract Algebra (IOAA*) curriculum (Larsen, Johnson, & Weber, 2013). IOAA is a research-based, inquiry-oriented curriculum that actively engages students in developing fundamental concepts of group theory and is designed for use in upper-division, undergraduate abstract algebra courses.

We analyzed 522 completed Group Theory Content Assessments (Melhuish, 2015) to investigate performance differences between students whose instructors implemented the IOAA curriculum (with support from the TIMES project), and those who instructors did not. Specifically we address the following two research questions:

1) What is the relationship between inquiry-oriented instruction, as manifested by the TIMES program, and student performance on a content assessment?
2) Is this relationship consistent across genders?

Based on the work of Freeman et al. (2014) we would expect to find a performance advantage for the students in the IOI classes. Further, given the similarities between IOI and IBL, we expect to see Laursen et al.'s (2014) findings replicated in our study – i.e., we expected to see more differences between the performance of women and men in a comparison group than in the IOI population. Confirmation of these hypotheses would corroborate research supporting active learning in general and inquiry-approaches in particular, whereas contradictory findings might provide insights into the differential ways that particular instructional approaches impact various populations.

Literature Review

The intention of IOI is to reposition students as central to the process of constructing and reinventing important mathematical ideas. Informed by the instructional design heuristics of Realistic Mathematics Education, IOI curricular materials leverage students' informal and intuitive ways of reasoning as starting points from which to build more sophisticated and formal mathematical understandings (Freudenthal, 1973). Specifically, the IOAA curricular materials include instructional units on groups and subgroups, isomorphism, and quotient groups. Each unit includes both a reinvention phase and a deductive phase. During the reinvention phase, students work on a sequence of tasks designed to help them develop and formalize a concept. Initial tasks in the sequence evoke student strategies and ways of thinking that anticipate the formal concepts. Then follow-up activities, and teacher guidance, leverage these ideas to develop the formal concepts. The end product of the reinvention phase is a formal definition and a collection of conjectures. The students then prove theorems that are typical of those found in other introductory group theory courses (Larsen, Johnson, & Weber, 2013). The cycles of inquiry and formalization, supported by the task sequence and guided by the instructor, are usually carried out in collaborative small-groups and whole-class discussions.

Research carried out prior to the TIMES project suggests that IOI in general, and IOAA in particular, has the potential to improve student learning by supporting the development of more robust conceptual understandings (e.g., Larsen, Johnson, & Bartlo, 2013, Rasmussen et al., 2006) and by improving student retention (Kwon, Rasmussen, & Allen, 2005), as compared to students from more traditional courses. These findings from IOI courses align with the meta-analysis of

Freeman et al. (2014), which found that across undergraduate STEM courses "student achievement was higher under active learning" (p. 8411). They also align with the findings of a study on one form of active learning in undergraduate mathematics known as Inquiry Based Learning (IBL). Laursen et al.'s (2014) work found that "students in IBL math-track courses reported greater learning gains than their non-IBL peers on every measure" (p. 409). Further, Laursen et al. found that IBL may be more equitable for women, reporting that, even with equivalent success rates in subsequent math coursework, "in non-IBL courses, women reported gaining less mastery than did men, but these differences vanished in IBL courses" (p. 415).

Laursen et al.'s (2014) findings are particularly relevant for our work because of the similarities between IBL and IOI. Laursen et al. (2014) characterize IBL as follows:

> …students construct, analyze, and critique mathematical arguments. Their ideas and explanations define and drive progress through the curriculum. In class, students present and discuss solutions alone at the board or via structured small-group work… (p. 407)

As this description is fairly consistent with (though more general than) the conceptualization of IOI adopted in the TIMES project, we had reason to believe that IOI classrooms would similarly support a "leveling of the playing field" for women and men.

That being said, there may be aspects of IOI (but not necessarily of IBL) in which the opportunities for student experiences, shaped by their interactions with their peers and their instructor, to create a dynamic that may negatively impact students from historically marginalized groups. For instance, implicit bias (Hill, Corbett, & St Rose, 2010) and stereotype threat (Good, Rattan, Dweck, 2012) may impact the ways peers interact during small group work. Furthermore, whole class discussions are shaped by instructor choices. Such decisions have varying implications for how different students may experience the class.

When considering the gendered experiences of students in collaborative classroom settings, there is reason to believe that these setting offer both affordances and constraints for women. Some literature suggests that classrooms emphasizing collaborative work, problem solving, and communication may be supportive for women (Du & Kolmos, 2009; Springer, Stanne, & Donovan, 1999). Moreover, there is research suggesting high school girls acclimate better than boys to learning environments that emphasize work on open-ended problems and conceptual understanding (Boaler, 1997; 2002). However, other research suggests that instructional approaches requiring students to develop their own problem-solving strategies may favor boys and men (e.g., Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Hyde and Jaffee (1998) offered a possible sociological explanation of such findings: the use of standard algorithms aligns with traditionally-valued feminine traits like compliance and meekness, whereas the use of invented strategies aligns with traditionally-valued masculine traits like confidence and independence. Research on the nature of social interactions in collaborative decision-making and facilitated discussions also offer insights into the way students may experience mathematics classrooms in gendered ways. Studies in non-mathematical collaborative settings have found that, when groups are tasked with arriving at a decision, women in groups made up predominantly of men spoke less and were interrupted more than men (Karpowitz, Mendelberg, & Shaker, 2012). Additionally, research indicates that during facilitated whole-class discussions in math classrooms students often receive qualitatively and quantitatively different opportunities to participate in ways that follow patterns of gender, race, and class (Black, 2004; Walshaw & Anthony, 2008).

In summary, Laursen et al. (2014) findings suggest that active learning approaches similar to IBL may have the potential to both improve student learning, and improve gender disparities, in undergraduate mathematics. However, the research literature also indicates that active learning classrooms have the to potential to reorganize the nature of classroom inequities – perhaps in ways that further marginalize historically under-represented populations. Our study has the potential to either corroborate Laursen et al.'s (2014) finding that active approaches like IBL can help eliminate gender disparity, or to problematize these findings and push us to more clearly articulate the conditions under which active learning classrooms are more equitable for various groups of students.

Methods

To investigate how IOI relates to student performance, we quantitatively analyzed data from 522 student content assessments. Of those assessments, 147 were completed by students of the TIMES fellows; the remaining 375 were from students in the national comparison sample. Here we detail the TIMES program, the instrument, our samples, and our analysis.

As part of the TIMES project, 13 mathematics instructors participated as abstract algebra TIMES Fellows. These fellows were provided support for implementing the IOAA curricular materials, which are formatted as task sequences that include rationale, examples of student work, and implementation suggestions. Due to documented challenges associated with implementing IOI (e.g., Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007), and IOAA in particular (Johnson & Larsen, 2012; Johnson, 2013), the TIMES Fellows were provided both prior and ongoing support. Summer workshops, held just prior to the instructors' implementation of the IOAA materials, had two main goals: to help the instructors develop an understanding of the curricular materials, including an overview of the mathematical development of the concepts; and to develop a shared vision of IOI, focusing on the roles of the teacher, the students, and the tasks (See Kuster et al., 2017). Online workgroups, held throughout the term in which the IOAA materials were being implemented, were hour-long weekly meetings with two components: an open forum devoted to addressing issues and concerns for the Fellows as they arose (e.g., facilitating group work, particularly difficult class sessions) and lesson studies. During the two lesson studies, the workgroup would first discuss the mathematics of the lesson, followed by a discussion of student learning goals and implementation considerations. After instructors taught the unit, they would share video-recorded clips of their instruction for group reflection and discussion. Throughout the sessions, the workgroup attended to the critical components of IOI – generating student ways of reasoning, building on student contributions, developing a shared understanding, and connecting to standard mathematical language and notation (Kuster et al., 2017).

The TIMES Fellows asked their students to complete the Group Theory Content Assessment (GTCA) (Melhuish, 2015). This assessment, developed to measure conceptual understanding of key concepts in group theory, spanned the topics of binary operations and their properties, group structures (including subgroups, quotient groups, and cyclic groups), element properties, and functions (homomorphisms and isomorphisms). The GTCA instrument was informed by textbook analysis and literature on student thinking and was designed to be applicable across a wide range of group theory courses. Instrument validation was achieved through a combination of expert review and multiple rounds of pilot testing (including clinical interviews) in which open-ended tasks were converted to a multiple-choice format based on student responses. (For specific details regarding the instrument development, see Melhuish, 2015.)

From the 13 TIMES Fellows, there were a total of 174 students, 147 of whom (84%) completed the GTCA. For our control, we have a national sample (Not-TIMES), with 375 students from 33 institutions. For Not-TIMES students we can presume (but not verify) that they did not experience IOI, as reports indicate that nationally the proportion of teachers using any form of non-lecture instructional approaches in abstract algebra is less than 10% (Keller, Johnson, Peterson, & Fukawa-Connelly, 2017).

Between the treatment (TIMES, n = 147) and control (Not-TIMES, n = 375) groups, we have a total of 522 participants: 275 students who identified as male, 240 as female, and 7 who otherwise identify or declined to identify their gender. The gender makeup was not significantly different (p = .229) between Not-TIMES and TIMES (48% and 42%, respectively, identified as women). We address our two research questions in stages. First, we investigate the relationship between IOI, then gender, and student performance on the GTCA via an exploratory univariate analysis. We calculated descriptive statistics and ran t-tests to look for evidence of performance differences on the GTCA between TIMES and NOT-TIMES students (with regard to Research Question 1) and to look for evidence of gender differences on the aggregate and within subgroups (with regard to Research Question 2).

The univariate analysis did provide evidence of significant differences when looking at the gender differences between the TIMES and Not-TIMES groups. Thus, in an attempt to control for compounding factors and to account for the nested structure of our data, we developed a Hierarchical Linear Model (HLM) to determine the robustness of the effects of IOI and interaction between IOI and gender. The appropriateness of a multi-level modeling approach for this data was determined by the sufficiency of the intraclass correlation (ICC) of the unconditional model (17%) and the results of the likelihood ratio test (χ^2= 38.368, p < .001) comparing the 1-level and 2-level null models.

As this was not a randomized treatment-control study, the inclusion of institutional nesting provides a means for accounting for differences between the TIMES institutions and the larger national sample. We conjectured that important institutional variables such as level of selectivity, 75^{th} percentile mathematics SAT scores (referred to as "SAT" for the rest of the paper), and Carnegie classification may account for performance differences on the GTCA. To test this, we developed an HLM model these variables as effects. Of these variables, only SAT was statistically significant; results indicating that a student at an institution one standard deviation above average would be estimated to score roughly half an item (0.564) higher on the GTCA (*p=0.034*). As a result, we incorporated normalized SAT as part of our model. Finally, we leveraged Snijders and Bosker's (2012) guidelines to determine our effect sizes on a Cohen's d (1988) scale, where effect sizes were calculated via looking at the cumulative effect of a variable of interest and dividing by the standard deviation of the control group.

Results

In looking for performance differences between students of TIMES Fellows as compared with the control group (i.e. Research Question 1), we see that TIMES students slightly outperformed Not-TIMES students by about half an item (6.64 vs. 6.21), but this difference is not statistically significant (t = -1.520, df = 520, p = .129). To investigate Research Question 2, we compared the GTCA performance by gender of the students in the two instructional groups (Figure 1). We found no significant difference in the Not-TIMES group where, on average, men outperform women by about half an item (p = .098). In the TIMES group however, men outperformed women by nearly 2 items on average (p < .001).

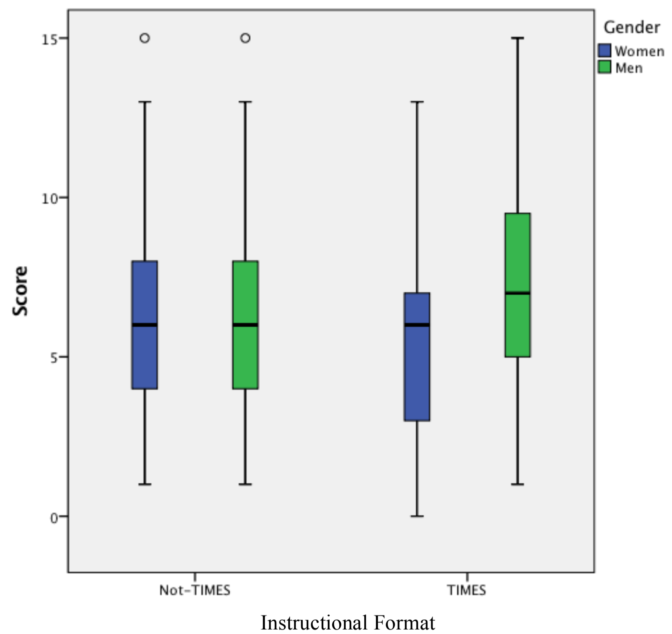

Figure 1. Gender Comparisons on GTCA Performance

Our initial univariate exploration provided evidence that there was no significant TIMES effect – i.e., TIMES students did not significantly outperform Not-TIMES students. However, this (non)effect of TIMES was *not* consistent across genders. While men in TIMES classes significantly outperformed the women in TIMES classes (and men in non-TIMES classes), this gender performance difference was not seen in the Not-TIMES classes.

Given the nested structure of the data, the univariate analysis does not rule out the possibility that these differences are better explained by differences in the instructor or by differences in the insitituion. Thus, we developed a series of HLMs to assess the robustness of the TIMES/Gender interaction effect. In revisting our first research question, this time controlling for instructor and SAT, we look at our simplified model. In this model, the estimated score for a TIMES student is 6.47 items while the estimated score for a not-TIMES student is 6.20 – a performance discrepancy between groups that is not statistically significant (p = .600). Thus, after accounting for potentially confounding variables, we again find no significant differences between the performance of TIMES and Not-TIMES students.

In revisting our second research question, again controlling for instructor and the inistutions' SAT, we look to the full model (see Table 1). This model verified that the interaction between gender and TIMES was robust and remained a significant factor (p = 0.014) even when nesting students within instructors, accounting for institutional differences in terms of SAT, and controlling for the global gender effect favoring men (p = 0.086). This model estimates that, for students at institution with mean SAT, a man in TIMES scores 7.23, a not-TIMES man secores 6.44, a not-TIMES woman scores 5.91, and a TIMES woman scores 5.86. So, while women are scoring roughly the same in TIMES and not-TIMES classes, men are scoring statistically significantly higher under the TIMES treatment.

Table 1
Model with TIMES and Gender Variables

	Coeff	SE	df	p
Intercept	5.913820	0.326461	67.321603	<.001
TIMES	-0.580915	0.600143	70.121264	0.336390
Level 1				
Man	0.526703	0.306206	458.653547	0.086089
TIMES*Man	1.372867	0.557205	453.700395	0.014115
Level 2				
SAT75	0.434603	0.225540	34.585700	0.062234
	Variance	SE		
Level 1 Residual	4757.087187	861.270931		
Level 2 Residual	1904.227364	1030.501765		

Discussion and Future Research

We found no difference in the performance of men and women in the national sample; however, under the TIMES treatment, a difference was present. Notably, this difference came from TIMES men outperforming Not-TIMES men, while the performance of women remained unchanged. While we see the detection of a gender performance difference within the IOI setting as an unfortunate finding, we are not arguing that the TIMES project, nor the implementation of IOI, is detrimental to women; in fact, both men and women under the TIMES treatment performed as well or better than students in the national comparison sample. However, the difference in learning outcomes between men and women among the TIMES population indicates that implementation of this curriculum is far from a guarantee of equitable instruction.

We suspect that there are important instructional differences between IOI and IBL that may impact different groups differently. This includes the routine use of student presentations in IBL classrooms (Hayward, Kogan, & Laursen, 2016), which are often distributed evenly across students and thus may remove barriers to equal participation; and the reliance on small-group work and whole-class discussions to develop the mathematical agenda in IOI, which may provide more opportunities for microaggressions and implicit bias to emerge. Indeed, preliminary analysis of 42 TIMES Fellows' instruction (across all content areas) suggests that, similar to the findings of Black (2004) and Walshaw and Anthony (2008), the TIMES instructors directed mathematically substantive questions at women at lower rates than men, and they re-voiced and elaborated contributions made by women at substantively lower rates than those made by men (Smith, Andrews-Larson, Reinholz, Stone-Johnstone, & Mullins, 2018).

We are hopeful that our future studies – investigating the gender performance difference we found in the IOI classes – will help us continue to refine our understandings of how features of student-centered instruction in undergraduate STEM can support robust student learning gains and equitable outcomes for all groups of students. It is our intention to use our findings to inform a critical examination of the effect of our interventions on the gendered experiences of our students and call on others in the field to do the same.

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