The Choice to Use Inquiry-Oriented Instruction: The INQUIRE instrument and differences across upper and lower division undergraduate courses

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In this study of mathematics teaching, we explore how to measure inquiry-oriented practices of college mathematics instructors. We offer a conceptualization of inquiry-oriented instruction organized by the instructional triangle (Cohen, Raudenbush, & Ball, 2003) and introduce an instrument developed to explore the extent to which elements of inquiry-oriented instruction are present in the teaching of university mathematics courses. This scale has been developed to explore what practices instructors currently use and eventually investigate the relationship between beliefs and practice. We show how we have operationalized inquiry-based instruction as self-report items and report preliminary findings that indicate our scales are performing well. We show that some inquiry-oriented practices are significantly more present in upper-division courses than lower-division courses. This suggests that at least some components of inquiry-oriented instruction are not reducible to individual differences (whether the instructor is an inquiry-based instructor), but also dependent in the context of instruction.

Keywords: Inquiry-oriented instruction, obligations, beliefs

College mathematics departments are faced with a growing need to develop innovative instructional practices to address the needs of increasingly diverse student bodies and declining numbers of mathematics majors (Holton, 2001; U.S. Department of Education, 2006). Even when instructors give a high-quality lecture, students often do not grasp the main ideas the instructor intends to convey (Goodstein & Neugbauer, 1995; Leron & Dubinsky, 1995; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016). Researchers have found that students learn better from active, student-centered instruction in college mathematics (Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006) and that meaningful participation in inquiry-based instruction has been linked to higher achievement and persistence for women and students of color (e.g., Boaler, 1997; Laursen, Hassi, Kogan & Weston, 2014).

Many varied interpretations of inquiry exist. For researchers seeking to understand inquiry-oriented instruction or instructors trying to implement it, we need a shared understanding of its components and to what extent those components exist among instructors that presently are trying to make their classes more active. This study seeks to investigate what inquiry-oriented practices instructors implement, as part of a project that seeks to understand also why they do it.

Literature and Framing

Studying Inquiry-Oriented Practices

Inquiry-oriented learning involves following the methods and practices of mathematicians (Yoshinobu & Jones, 2012) or getting students to engage in "authentic mathematical activity" (Johnson, Caughman, Fredericks, & Gibson, 2013). Many studies have focused on the design and implementation of inquiry-based curriculum, such as with the linear algebra magic carpet ride task (Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012). In that study, Wawro et al. (2012) designed a task that allowed students to discover the concepts of span, linear independence, and linear independence and invent their own definitions of those concepts. Other studies have documented a handful of challenges that instructors face during implementation and how those challenges can be overcome. For example, Yoshinobu & Jones (2012) wrote about the coverage issue: that many instructors resist using inquiry-oriented practices because they fear they will not have time to cover the content their institution expects. No studies known to the authors of this paper attempt to document what various inquiry-oriented practices are being taken up by college instructors on a large scale. Some challenges of such a study are to identify the components of inquiry-based instruction that might be documented and to design a feasible method to study the practices of a large enough sample. Before subscribing effort to rate instructors using observational methods, it would be useful to do descriptive work in which one can study the feasibility of any such rating. A survey using self-reports is an economical way to discover and identify those components.

The Instructional Triangle

The instructional triangle (Cohen, Raudenbush, & Ball, 2003; Ball & Forzani, 2007) is a useful frame to organize the study of instruction and, in particular, to organize our identification of possible indicators of inquiry-based instruction. The instructional triangle is composed of the interactions between the teacher, students, the content, and the environment surrounding all three. The triangle suggests the situatedness of these interactions in environments in four possible dimensions of instructional actions (see Table 1).

The first dimension addresses how the students are given opportunities to engage with the mathematical content, for example, the extent to which they engage in authentic mathematical work. Education researchers have stressed the importance of students' school experiences aligning with the disciplinary practices of scholars across any subject (Bruner, 1960; Dewey, 1902; Schwab, 1978). The work of mathematicians involves contributing ideas, struggling with definitions, experimenting with examples, proposing conjectures, propositions, and theorems, and providing proofs and arguments for those claims. Lakatos (1976) wrote in *Proofs and Refutations* about an imaginary classroom dialogue surrounding the problem of finding a relation between the number of vertices, edges, and faces of polyhedra. As the class progresses, they consider examples and counterexamples, and construct various conjectures and proofs. The process is anything but linear and conjectures are continually revised as students encounter new evidence and arguments posed by each other. This exploratory discovery has been cited by mathematics educators to explain what it means to think mathematically (e.g., Schoenfeld, 1992). That text shows by example that doing mathematics not only involves solving problems, but also formulating hypotheses from observations and *problem-posing* (Silver, 1994, 1997).

The second dimension deals with how the instructor relates to the students vis-a-vis the knowledge to be learned, including, for example, how much they involve students in the development of new knowledge. Gonzalez (2013), a practitioner of inquiry-based learning, described his role as becoming more of a "'guide on the side' than a 'sage on the stage'" (p. 35). When a student is stuck, the instructor does not give the solution away, but helps by posing a question, getting other students to help, or finding a smaller problem or special case that can help them make progress on the larger problem (Yoshinobu & Jones, 2012). While strict lecture with the instructor dictating the lesson in the front of the classroom may be thought of as providing the least opportunities for inquiry, there are ways to make lecture more responsive. Burn, Mesa, and White (2015) used the term *interactive lecture* to refer to presenting material in an engaging way that included questions and answers.

The third dimension considers whether and how students have interactions with their peers as they engage in mathematical work. In inquiry-oriented classrooms, students are often asked to present their solutions to classmates and receive feedback on their reasoning (Gonzalez, 2013; Hayward, Kogan, & Laursen, 2016; Laursen & Hassi, 2010; Yoshinobu et al., 2011) either at the front of the classroom or in small groups (Yoshinobu & Jones, 2012). In small group discussions, students often work on problems together, while during presentations, one student leads the class in finding a proof or solution and other students can comment or ask questions. Practitioners like Renz (1999) report that when students interact, they theoretically gain motivation from their peers to check their own work carefully and present their ideas clearly.

Table 1. *Conceptualization of inquiry-oriented instructional practices*

Triangle Relationship	**Constructs**	**Description**
Student-Content	Open problems	Posing problems that either have multiple solutions or multiple nontrivial ways of arriving at a solution
	Constructing	Posing tasks that ask students to make conjectures and construct arguments
	Critiquing	Asking students to critique the reasoning of themselves and others
	Definition-formulating	Inventing or reinventing mathematical definitions
Teacher-Student	Interactive lecture	Incorporating interaction to whole class lecture: Requesting feedback from students, asking questions of students, and having students engage with the mathematics during lecture
	Hinting without telling	Guiding a student to work productively without directly telling the student a correct way to proceed
Student-Student	Group work	Creating an environment where students work together on mathematical tasks or problems
	Student Presentations	Having a student or students present completed or in-progress work to the class
Teacher-Content	Class preparation	Planning lessons to intentionally contain opportunities to engage in inquiry-oriented learning around the content being taught

And finally, the fourth dimension addresses how the instructor engages with the content, for example, through exploring the content on their own or designing resources meant to engage students in discovery. Instructors engage with the mathematical content to design and choose the inquiry-oriented problems or activities for their students (Gonzalez, 2013). An instructor's mathematical content knowledge is a prerequisite to this work (Wagner, Speer, & Rossa, 2007). An icon of the inquiry-based instruction movement has been the mathematician R. L. Moore, famous and infamous for his method of teaching students by engaging them in problems (Parker, 2005). Moore engaged with the content by understanding the mathematics and his students well enough to assign problems that were challenging enough to instill perseverance and pride, but not so challenging that students would grow discouraged and give up (Mahavier, 1999). We have attempted to review the literature to include inquiry-oriented practices within each of these categories, though we do not include a comprehensive literature review due to space limitations.

Instead of taking the simplistic view that some instructors implement inquiry-based learning and some do not, our multidimensional conceptualization of inquiry-oriented instruction allows researchers to anticipate various elements that might be present in some classes and not in other classes. We avoid rushing to a synthetic statement that inquiry-oriented instruction is one single thing and instead seek to examine whether we can identify some of its components and use them to characterize variability in the practices that present themselves as inquiry-oriented. Organizing the numerous components of practice around the instructional triangle allows us to measure the extent to which various characteristics of inquiry-oriented instruction are present.

With this framing in mind, we ask the following research question: How can we measure the inquiry-oriented practices of college mathematics instructors? In this paper, we begin exploring preliminary trends in the data collected in response to a survey that operationalized those four dimensions.

Methods

Instrument Design

The inquiry-oriented instruction review (hereafter, INQUIRE) instrument contains 62 items split into the constructs described in the framing. Each item reflects a literature-based inquiry-oriented practice which the participant can respond to on a Likert-type scale from 1-Never to 6-Multiple times per class. See Table 2 for an example item from each construct and Appendix A for additional examples. Cognitive interviews for the INQUIRE items were conducted with five mathematics doctoral students, four mathematics education doctoral students, and one mathematics department faculty member, all from two midwestern Research I universities. All had at least three years of teaching experience at the college level.

Sample

For recruitment, we used a comprehensive list of Research I mathematics departments in the U.S. We emailed the call for participants to each mathematics department and requested that they forward it to their instructors that had a minimum one-year teaching experience. Though many participants have completed the INQUIRE instrument (*N=247)* here we report the characteristics of participants that have completed both lower-division and upper division sections of the survey (*N=69*). This narrows the sample because many instructors, especially graduate students, have not taught upper-division courses.

Our sub-sample[1] consisted mostly of graduate student instructors (*N=20, 29.9%)* and non-tenure-track faculty (*N=14, 20.9%)*. The remaining instructors were postdoctoral fellows (*N=12, 17.9%)*, tenure-track faculty *(N=11, 16.4%)*, or tenured faculty (*N=9, 13.4%)*. The mean experience teaching was 9.14 years (SD=7.14). There were 34 males (50.8%), 31 females (46.3%), and 2 chose not to specify. There were 21 (31.3%) instructors who claimed to use inquiry-oriented or inquiry-based instruction, 19 (28.4%) who claimed to not use it, and 27 (40.3%) that either had not heard of it or were unsure.

[1] Based on *N=67* of the *N=69* participants, due to two participants not completing the background survey.

Table 2. *Examples of items in the INQUIRE instrument for each construct*

Triangle Relationship	Constructs	Example Item
Student-Content	Open problems	How often do you task students with problems where there are multiple solutions?
	Constructing	How often do you ask students to generalize a claim?
	Critiquing	How often do you provide students with arguments for them to critique?
	Definition-formulating	How often do you ask students to revise a definition?
Teacher-Student	Interactive lecture	While teaching the whole class, how often, after demonstrating how to solve a problem, do you ask students to try a similar problem?
	Hinting without telling	If a student asks you to look at his or her work, how often do you respond without evaluating whether or not it was correct?
Student-Student	Group work	How often do you have students work together in groups?
	Student Presentations	How often do you have students present work to the class?
Teacher-Content	Class preparation	How often do you design a sequence of problems so that students will discover something?

Results

The reliability fit statistics for item grouping in the INQUIRE instrument are satisfactory. A common cutoff for Cronbach's alpha is to consider values over 0.7 as acceptable and those below 0.5 as unacceptable (Kline, 2005), and inter-item correlations (IICs) should range between .15 and .50 (Clark & Watson, 1995). All item groupings had at least acceptable alpha scores, showing good internal consistency as shown in Table 3. Four IICs were too high (lower-division presentations and upper-division presentations, critiquing and group work), indicating that the items associated with those questions may be too similar. We can remedy this issue for creating scores later by removing some extra items.

The descriptive statistics from the INQUIRE instrument are shown in Table 4. We conducted a paired sample two-tailed t-test for each construct, results also shown in Table 4. For many of the categories, instructors report engaging students in significantly more inquiry-oriented practices in upper-division courses than lower-division courses. The only practices that were not practiced more in upper-division courses were interactive lecture, group work, and class preparation. For instructors newly attempting to implement inquiry-oriented instruction, these areas might seem more feasible or accessible.

Directions for Future Research

This study offers a method to study the inquiry-oriented practices on a broad scale. As more instructors attempt to implement more innovative practices, it could be useful for informing mathematics departments, inquiry-based-learning centers, or other stakeholders to understand what practices are currently used nationally and what factors predict their use. For our study, our first steps with the INQUIRE instrument will be to conduct a factor analysis to refine the items in our scale. The INQUIRE instrument is one of five instruments completed by all participants. We then will use methods from classical test theory and structural equation modeling to understand the relationships between beliefs, professional obligations (Herbst & Chazan, 2012), and the inquiry-oriented practices of college mathematics instructors. Early analysis has shown indications that beliefs do predict practices, but for some practices, professional obligations improve the model. We intend to continue investigating what inquiry-oriented practices can be better explained with a social lens in addition to an individual lens.

Table 3. *Reliability Statistics*

Relationship	Constructs	Lower-Division		Upper-Division	
		IIC	α	IIC	α
Student-Content	Solving open problems	.44	.79	.33	.71
	Constructing	.45	.85	.31	.76
	Critiquing	.39	.81	.56	.90
	Definition-formulating	.43	.79	.33	.71
Teacher-Student	Interactive Lecture	.25	.73	.36	.82
	Hinting without telling	.45	.77	.48	.82
Student-Student	Group work	.41	.87	.70	.95
	Presentations	.54	.89	.59	.93
Teacher-Content	Class Preparation	.29	.79	.35	.83

Table 4. Mean, standard errors, and comparison test results for the INQUIRE instrument *(N=69)*

Relationship	Constructs	Lower-Division Courses	Upper-division Courses	Difference
Student-Content	Solving open problems	3.21(0.13)	4.56(.14)	-1.36(.15)***
	Constructing	2.52(.13)	4.04(.16)	-1.52(.11)***
	Critiquing	2.42(.12)	3.93(.16)	-1.51(.13)***
	Definition-formulating	2.43(0.13)	3.78(.16)	-1.34(.12)***
Teacher-Student	Interactive Lecture	4.44(.09)	4.28(0.10)	.16(.75)
	Hinting without telling	3.39(0.08)	4.16(.98)	-0.76(.10)***
Student-Student	Group work	3.32(.16)	3.47(.17)	-.14(.13)
	Presentations	1.83(.12)	2.22(1.17)	-.39(.12)**
Teacher-Content	Class Preparation	3.83(.10)	3.90(.12)	-.07(.12)

**p<.05, **p<0.01, ***p<0.001*

Appendix A: Sample INQUIRE items

Student-Content Interaction

1. How often do you ask students to propose a definition?
2. How often do you ask students to construct mathematical arguments (e.g., justifying a solution or claim)?
3. How often do you give students problems that can be solved more than one way?
4. How often do you give students a sequence of tasks to solve that will lead them to discover something?
5. How often do you ask a student to find an error in a finished proof or solution?

Teacher-Student Interaction

6. While teaching the whole class, how often do you make an effort to elicit questions from students (e.g., by having them fill out exit slips, use clickers, giving them time to think of questions they might have, etc.)?
7. While teaching the whole class, how often do you pause your presentation to ask students to work on a problem or problems?
8. While you are solving a problem or constructing a proof with the whole class, how often do you ask students for suggestions of what to do next?
9. If a student is stuck on a problem and asks for help during class, how often do you give them a hint on how to proceed?
10. If a student is stuck on a problem and asks for help during class, how often do you help them by reminding them of an approach or strategy they've already learned?

Student-Student Interaction

11. How often do you have students give feedback to student-presenters?
12. How often do you ask a student to study and present a new topic to the class?
13. How often do you have students discuss a problem with each other?
14. If a student asks a question, how often do you redirect the question to other students?
15. How often do you encourage students to question each other's reasoning?

Teacher-Content Interaction

16. How often do you prepare worksheets for students to work on during class?
17. How often do you search in textbooks (including the one you're teaching from, if you are) or other resources to find material that will help students learn the course content?
18. How often do you design or search for problems or activities that aim to guide students to discover something you want them to learn?
19. How often do you design your lesson to include experiences you have had learning mathematics?
20. How often do you design your lesson to include experiences you have had doing mathematics?

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Erratum

We found a survey implementation error that caused a nonrandom portion of our sample to skip the remainder of the survey. If participants selected the responses “1-Never” to the question, “How often do you ask students to revise a definition?” they were skipped past the remainder of the survey, including the lower-division student-student, teacher-content questions, and the upper-division questions. Thus the statistical significance we reported in Table 4 of increased inquiry in upper-division courses may be due to the systematic missingness from the participants that took those items.