Calculus Variations as Figured Worlds for Mathematical Identity Development

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Calculus is often an essential milestone during a students' time in college and can be especially impactful for students wishing to major in in a math or science field. Given its relative importance, the ways in which calculus courses are delivered can have a lasting impact on a student's trajectory and relationship with mathematics. In this study we document the ways in which three calculus course variations at the same University operate to promote different mathematics identities for students. Drawing on the Holland et. al.'s (1998) framework of figured worlds we showcase the ways in which these course variations act as if they are different calculus worlds that constitute socially organized and produced realms of being. We highlight the ways in which these figured worlds position or fail to position students with the opportunity to refigure themselves and others as learners and doers of mathematics.

Keywords: Calculus, Math Identity, Figured Worlds, Course Variations

In the United States there is a national movement to increase the number of awarded STEM degrees in order to address the nearly 1 million additional STEM degrees needed to support the nation's growing research and technology economy (PCAST, 2012). Among the recommendations to address this need, the PCAST report recommended the adoption of empirically validated teaching practices, replacing standard lab courses with discovery-based research courses, addressing the mathematics-preparation gap, and diversify pathways to STEM degrees. Additionally, any efforts to improve the quality of undergraduate STEM education must also attend to fostering an environment that promotes diversity and inclusion in STEM classrooms (National Academies of Sciences Engineering and Medicine, 2017).

The vision and enactment of creating an equitable robust STEM education is a complex and multifaceted endeavor that will require continued research; however, one such promising approach in undergraduate mathematics that has been identified with successful calculus programs is the tailoring of calculus courses to meet the needs of individual students, which we refer to as course variations. Course variations have the potential for addressing the recommendations from the PCAST report since they can specifically address the preparation gap for students by incorporating prerequisite material in courses or stretching out the course content, can infuse labs and standard based teaching in courses tailored for science majors, and even provide diverse pathways into STEM for those that have taken a non-traditional math background through transition courses. Author (2016) documented how these variations to the standard course across the US have been associated with greater rates of passing calculus and put forth a call to future research to examine the ways that these courses may help promote a sense of community and identity development among students in the different variations. We take up this call for future research in this report, by examining *how the structures and activities of three different calculus courses at the same undergraduate institution impact the types of possible mathematical identities that emerge from those contexts?*

Framework and Literature Review

Mathematical identity has become a central and powerful concept in the analysis of mathematical learning, in part due to the recent social and political turn in education (Gutiérrez, 2010). Identity frameworks in math education have drawn largely from sociocultural perspectives that link identity and learning to one another and arise from social practices. Additionally, this research often utilizes positioning theory to account for identity as constructed through social interactions to construct storylines about who a person is in relation to others in a social context (Langer-Osuna & Esmonde, 2016). Holland, Lachicotte Jr., Skinner, and Cain (1998) sociocultural theory of identity and self, known as figured worlds, is useful as a tool for studying identity production in education, and how the context of education allows or does not allow the emergence of certain identities. Figured worlds are "socially and culturally constructed realms of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al.,1998, p. 52). Figured worlds are dynamic. They are constantly formed and re-formed in in relation to the everyday activities and events that occur within the realm of possible "as if" worlds. Figured worlds are thus situated in a social context and time period, and represent a reflexive relationship and negotiation of the possible identities that can be constructed and affirmed in the figured world. As cited by Urrieta (2007), Holland claims that figured worlds have four characteristics:

(1) Figured worlds are cultural phenomenon to which people are recruited, or into which people enter, and that develop through the work of their participants.
(2) Figured worlds function as contexts of meaning within which social encounters have significance and people's positions matter. Activities relevant to these worlds take meaning from them and are situated in particular times and places.
(3) Figured worlds are socially organized and reproduced, which means that in them people are sorted and learn to relate to each other in different ways.
(4) Figured worlds distribute people by relating them to landscapes of action (personae); thus activities related to the worlds are populated by familiar social types and host to individual senses of self.

Boaler and Greeno (2000) draw on the concepts of figured worlds to illustrate how two different types of high school classrooms afforded students different identities and storylines of mathematical learners. One such figured world of mathematics classroom, drew on the concept of "received knowing" and promoted a concept of doing mathematics as memorization and being able to quickly recall information. In contrast, the other figured world of mathematics classroom drew on the concept of "connected knowing" and promoted a concept of doing mathematics as making sense of mathematical concepts and procedures. The "connected knowing" world promoted as sense of agency among the mathematical learners since they were encouraged to draw on their own interpretations of mathematics to make sense of the concepts. Boaler and Greeno's study highlights how the context of education setting and approach to teaching impact students' identity production as learners and doers of mathematics, which impacts their choices for continuing or dropping out of further engagement with mathematics. This is especially problematic since there is some indication that educational contexts which promote decontextualized and abstract knowledge are more alienating for women and non-western students and therefore hindering the goals of educational equity in STEM.

Solomon, Croft and Lawson (2010) examined how mathematics support centers, which were intended to support skill development for engineering students, were dynamically co-opted by the students to support the development of group learning strategies which promoted a strong community identity among the participants. This study highlights the way the undergraduate STEM community of practice, which can often be highly competitive and individualistic, can refigure itself by reflecting on the positional identities that can be challenged in that space by drawing on the physical resources and artifacts to disrupt the available storylines. For instance, the physical space of the tutoring center, allowed students to refigure their relational identity to mathematics as a social endeavor of helping each other succeed.

Methods

The analysis presented draws on student focus group data from one University, which we refer to as Tree Line University (TLU). TLU offers three different calculus courses. In addition to the standard offering, TLU has a coordinated calculus-physics course for advanced students and a life science course, which includes a focus on biology. For each of these three calculus offerings, we conducted a focus group with three to five students currently enrolled in the target course. We use focus groups for this research since they help highlight the nature of figured worlds which are socially constructed among the members. Students for the focus group were sent an email invitation as well as recruiting efforts done in-person during the course. As such the students who participated represent a self-selected sample of student willing to participate in the focus group, and while this may not be representative of the entire course they highlight the realm of possible mathematical identities afforded in each of the courses. The focus group conversations centered on topics such as who they are, their experiences in the course, how and why they chose this particular course, what happens during a typical class period, how they relate to others in this course as well as to students in different calculus courses. All focus group interviews were audio recorded and transcribed for subsequent, thematic analysis (Braun & Clarke, 2006). Guided by our theoretical framing of figured worlds, we developed narrative accounts in a collaborative endeavor among the researchers by first producing a descriptive account of the focus group and then using within and cross-case comparison to develop themes related to the research focus. These narrative accounts centered around the themes of students' emerging mathematical identities, sense of community or belonging, and positional relationship to calculus as (ir)relevant to their major and career goals. We present how these themes are enacted as figured worlds in each of the three calculus courses along with illustrative quotations from the narrative accounts.

Results

Calculus for Life Sciences: A refiguring of productive mathematical identities

Calculus for Life Sciences at TLU functions as a combined differential and integral calculus course without topics in trigonometry. The course was originally designed at the request of the college of life sciences and agriculture for students majoring in the life sciences. The content remains fairly similar to the standard calculus course but has what faculty described as a "lighter approach" that emphasizes concepts and some application of topics. Our focus group in this course included five students enrolled with the same instructor (Dr. B) for the lecture session but who had different teaching assistants for the twice weekly recitation sections.

Many of the students in the focus group conveyed that prior to enrolling in this course they had identities as poor performers in mathematics, which made them anxious to take a university calculus course. One student shared that they had taken precalculus and had gotten a C- in the course, and stated that it, "was the lowest grade I had ever gotten for a college class," and as a result was worried about how well they would do in this class. Several of the other students in the focus group concurred with this sentiment, with one student stating, "I did so poorly in that class, and I just thought like I am not meant to pass calculus." Other students discussed how the gap between their last math course in secondary school and taking this calculus course made them less prepared, and that they were "nervous going into calculus." Students in the focus group had a personal social history (history-in-person) that positioned them outside of the world of learners and doers of mathematics. For example, one student stated that they were, “someone who is not naturally inclined to math,” while another stated, “I am not meant to pass calculus.” However, as we will show, the students conveyed that through their experiences in this course, they were able to refigure their identities as productive mathematical learners and doers largely as a result from positive interactions with their instructor.

Students in the focus group conveyed that as a result of this course they now viewed themselves as someone who was capable of learning and doing mathematics. One student said that “I feel like I'm not completely hopeless at all in math anymore.” This sentiment was supported by several of the students who recognized a shift from their prior conceptions and experience in mathematics. For example, one student said, “I can actually do this, rather than like, in many past courses where I really have no idea what's going on." Students discussed how they were really “understanding” what they were doing rather than memorizing formulas, which aligned with the goal and vision of the course from the faculty perspective. As a result, students were able to refigure their positionality towards learning mathematics, as exemplified in the following quotes: “I'll be able to succeed in other math heavy courses” and it “boosts my confidence in that regard.”

One of the contributing factors that helped students refigure their mathematical identities was their relationship with the instructor. “I can't say enough about our professor, this is probably the only math class that's really felt like it made sense in my life.” Students described instructional practice that contributed to their positive experience such as the teacher breaking down concepts in a way that made sense, using anonymous polling to see how they were feeling about course concepts, and providing prerequisite information such as the quadratic formula without assuming the students had memorized this information. These practices seemed to convey to the students that the instructor cared about them and their learning, allowing for them to acknowledge their past mathematical identity while being supported in the negotiation of productive mathematical identities. The impact of the individual instructor versus other features can't be isolated in this study; however, the instructor through instructional techniques allowed for the enactment of a figured world that aligned with the goals of the course to have students focus on understanding and connected knowing.

There were also ways in which the enactment of the course variation positioned the students outside the world of mathematics learners. For instance, while some of the students mentioned that they were unware of the difference between calculus for life sciences and the standard calculus course, some of them mentioned the ways in which it was “low base calculus” or “more basic algebra” compared to “real calculus.” One student even described how their friend who was studying physics teased them saying, “you're not taking calculus, calc for life sciences is just like classical math.” Additionally, many of the students felt that the stated goal of the course to serve life sciences students was too broad. This resulted in students feeling that the course was not tailored to their specific discipline identities, “I'm either getting pushed aside or pushed under the rug with everybody else by just saying, “Oh well, you're in the life sciences major, you got to do this.” In this figured world of calculus for life sciences, students were maintaining a strong discipline identity (equine science, zoology) which they viewed as not needing calculus.

Honors Calculus: A collaborative community of academically-minded students

Honors calculus at TLU is a unique course that it is designed to integrate topics in physics and calculus and takes a theoretical approach to the material. Our focus group consisted of three students majoring in mathematics. Students emphasized the difficulty of this course by the fact that they often have to rely on one another to finish the homework and study for exams. For example, an agreed upon sentiment is that “Collaboration is actually one of the strengths of the class…you know everyone in the class, you feel like you can trust that they're going to put in the effort, and you're going to put in the effort, and you're going to come together if you need to.” The word “trust” was often used by the students in this focus group interview. They felt that there was a need to trust each other in order to do well. It is important to note that the objective for these students was clear; it was not to just pass the class, but to do well in the course together.

All of the students in this focus group had AP calculus credit. They entered into a world where they viewed their peers as equals who enjoyed learning and doing mathematics as much as they did. From the start they described a course that positioned them in the figured world of calculus where they felt accepted and academically challenged. This is reflected by the students' frequent reference to being surrounded by people who are the “same.” One student in the focus group reinforced this idea as follows: “In my calculus class, we have students who are all STEM. They are students who have the same mindset”. These students are in a space where they are comfortable to acknowledge that they are joined by, “intelligent people who have the same common objective”. This highlights how mathematical identity can be formed when students are surrounded by people who they perceive to be cut from the same cloth.

Students were able to relate to each other and work together based on the fact that they are all coming in with similar interests, similar class objectives and career goals. During lecture, they were required to work in groups, which was a point of contingency at the beginning of the semester. There was reluctance from some students to work with one another because they wanted to “motor through” the activities. However, once they created a world where they were able to share their ideas they came to view group work positively. As one focus group member put it, “I get to share my perspective, I get to hear their perspective,” which they felt created a class that was more enjoyable. The figured world of honors calculus that the students created for themselves allowed them to grow and form a mathematical identity that centers around succeeding, understanding the material, and supporting others.

In this figured world, full of high achieving STEM majors, the students in the focus group reported having an extremely strong sense of community. One student explained how close knit they are as follows: “If I have a concern about anything really, I feel like I can go and find someone from the class and talk to them about it and ask them what they think. And, you know, that's something that I think might be more exclusive to the [Honors Calculus].” An important aspect to highlight regarding their community is that the students in the focus group felt a closer sense of community in the honors calculus class than they did in any of their other honors classes. One contributing factor as the students described it was the focus of the class being all STEM majors, who had a similar interest, didn't “wince” at the word math, and had high academic engagement.

Standard Calculus: A realm of disconnected knowing and isolation

The standard calculus sequence at TLU is primarily a service course for engineering majors. The focus group consisted of three men, with majors in ocean engineering, mechanical engineering, and chemical engineering. The students in the focus group had varied secondary school mathematics experiences where one student took a non-AP calculus course, one took an AP calculus course, and the third student did not take calculus in secondary school.

The one student who had not taken calculus in secondary school described Calculus 1 as “fast-paced” and not well-organized. He also expressed some personal disconnect with the material when he said, “I didn't know what a derivative, like what is the definition of a derivative, till like two weeks after we had started them." The other two students who had taken calculus in secondary school also felt that the course was fast paced but were less concerned with the material. In general, the three students positioned themselves as external to calculus, where calculus was something they had to do, as opposed to something that they were excited about learning. For example, one student said, "it's a class and I have to do work for it. That's just normal college stuff" and another student said calculus was a course "they had to take." Thus, upon entering calculus as first year students, none of the three positioned themselves as particularly excited about mathematics or very interested in mathematics. As they progressed from Calculus 1 to Calculus 2, this feeling of being disconnected from mathematics was *not* refigured, but rather seemed to become entrenched and reified.

In both Calculus 1 and 2, the three students had similar experiences in lecture. One student explained that he felt so disconnected that he stopped going to his assigned lecture and attended a different lecture instead. He recounted that in class he felt, "nobody knows what's going on because you're just up there writing, and you won't answer the questions. So, this is very frustrating." Another student chimed in that "Everything that he just said that happens this semester, happened for me last semester." The feeling of being personally disconnected from their instructors and the course content was amplified in Calculus 2. In contrast to Calculus 1 where they felt the material was more applicable and useful, their experiences in Calculus 2 was on memorization. For example, one student contrasted his experience in Calculus 1 and 2 as follows: "The expectation [in Calculus 1] was that there would be understanding. The latter [Calculus 2] is memorization without any expectation of understanding." This was a common sentiment for all three students. In fact, one student explained that he was told that Calculus 2 is "really advanced math" and so there they are not expected to "understand what we are doing." Even his teaching assistant (TA) positioned the content as something that was not within their reach for understanding. "And like my TA has dropped a line similar to just saying like, 'You don't need to know further, this is what you need in order to do this. So, this is what you're given.'" Thus, their experience in calculus at TLU resonates with the figured world of "received knowing" described by Boaler and Greeno (2000).

TLU's no calculator policy seemed to further figure calculus as something that is disconnected from their interests and previous experiences. For example, one student explained that the no calculator policy in calculus stood in contrast to how he imagined his future self in the workplace as an engineer. "You're not going to be working in a laboratory somewhere and they're just having you do calculations derivatives and integrals like, in your head. Like you're going to have a calculator. Especially if you want to do real-world problems." They also contrasted their calculator experience in calculus with that in physics and chemistry, where calculators are used all the time. This positioned mathematics for them as outside the realm of connection with other disciplines.

When asked about the extent to which they felt they had formed bonds or connections with their classmates, the three students agreed that any relationships they formed were *not* the result of how class was structured or due to any effort on the part of their instructors. Instead, those that they do homework with are either friends or live in the same residence hall. Their ability to work with a wide range of students from different lectures was made possible because TLU has tightly coordinated curriculum, homework, and assessments. As these three students explained, "there's a lot of behind the scenes learning from kids explaining, or students explaining stuff to one another" and "there's a lot of, frankly, bonding over freaking out." Thus, at a system level, the course coordination allowed for considerable peer to peer bonding that otherwise might not have happened and allowed these three students to refigure their relational identities as helping residence hallmates survive calculus.

Discussion

Given the exploratory nature of this work we did not posit any hypothesis regarding how the different course variations would impact student mathematical identities, and instead our aim was to capture the salient features described by the students and how those related to their beliefs about knowing and doing mathematics. The enactment of these figured worlds considers the totality of the lived experience such as the role of the instructor, calculator policies, discipline-based problems, and the structures surrounding entry and pathways into the courses. These elements cannot be separated since they are fundamentally tied together. For instance, instructors for the calculus for life sciences are selected knowing the course should emphasize mathematical understanding and are aware that most of the students have had negative experiences with mathematics prior to starting the course. This results in assigning instructors who often are more student-centered in their teaching approach.

Comparing across the three variations in the study, we can see the ways in which the four characteristics of the figured world, as previously defined in the theoretical background section, contributed to the productive mathematical identity, mostly for students in the calculus for life sciences and the honors calculus. For instance regarding the first characteristic, the ways in which students were recruited or enter into the figured world, this helped promote a sense of academic-excellence and equivalency among the students in the honors sections because they had been invited to enroll in the course as part of the honors program as compared to students in the standard course which were “forced” to take the course as part of a degree requirement. Looking at the second characteristic, one can see how the social interactions and positionality of the instructor, Dr. B, for the life science students gave meaning to their sense of being cared for as a calculus student, allowing them to align and refigure themselves with positive mathematical interactions. Over the course of the semester, students in each of the three variations were socially organized, in some cases as peers actively supporting one another, and others as students living in the same residence working to survive in the figured world. Additionally, the fourth characteristic arose during students’ understanding of familiar social types and having a strong individual sense of self that was related to a discipline identity such as an engineer in the standard course or a zoologist in the life science that positioned the use of calculus as ancillary to their pursuits.

The course variations at TLU served as figured worlds for the students that seemed to impact their mathematical identity. The role of the instructor to either express care for their learning, to encourage peer collaboration, or to lecture the material at a quick pace was a paramount factor in how the students described their beliefs about being able to learn and do mathematics. The way the instructors approached teaching we speculate is tied with the programmatic features of the course variation. Whereby the standard course is content heavy and puts pressure on instructors to cover the material through lecture, the honors course has more contact hours and was designed with collaborative labs, and the life science course focus on understanding and less on computation which promotes instructor inquiry into students thinking.

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