Professors Intentions' and Student Learning in an Online Homework Assignment

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*Homework accounts for the majority of undergraduate mathematics students' interaction with the content. However, we do not know much about what students learn from homework. This paper reports on a pilot study of why professors chose particular homework problems, what they hoped students would learn from them, and whether students' engagement with the problems reflected those outcomes. Results show students gained the desired familiarity with notation and procedures. The results also speak to how professors manage the content between what they discuss in class, homework problems, and intentional overlap between the two.*

*Key words: online homework, sequences, instructional triangle, calculus*

## Background and Theoretical Perspective

One way researchers have conceptualized mathematics instruction is as "interactions among teachers and students around content, in environments" (Cohen, Raudenbush, & Ball, 2003, p. 122). In this perspective we can think of instruction as a triangle that relates the teacher, the knowledge at stake (content), and the student (Figure 1). The student vertex includes the mathematical tasks students work on and the milieu in which they experience those tasks. A milieu is a "counterpart environment[s] that provides feedback on the actions of students" (Herbst & Chazan, 2012, p. 9). The interactions among the vertices are governed in part by the didactic contract (Brousseau, 1997), the set of implicitly-negotiated expectations between teachers and students. For example, students expect professors to provide opportunities to learn the knowledge at stake. Professors expect students to do the tasks (or other activities) that represent learning opportunities. At the undergraduate level, an important component of the didactic contract is the expectation that students will spend significant time out of class interacting with the knowledge at stake (Ellis et al., 2015).

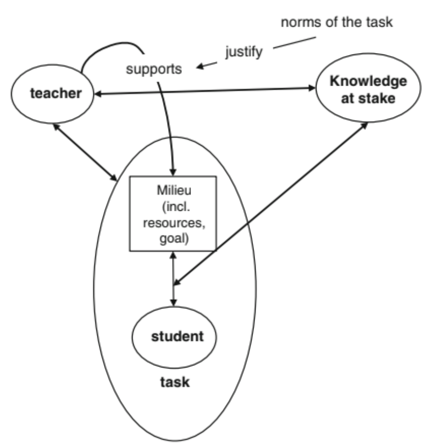


*Figure 1. Instructional triangle (Ellis et al., 2015, p. 270; Herbst & Chazan, 2012, p. 10).*

Homework represents the primary milieu for students' out-of-class learning. University calculus I students spend more time doing homework than they do in class (Ellis et al., 2015;

Krause & Putnam, 2016). As such, homework accounts for the majority of students' interaction with mathematics content and mathematics tasks (White & Mesa, 2014). White and Mesa (2014) found instructors view homework in general as a way for students to learn through repetition, understand algebraic manipulations, and apply mathematics to realistic situations. However, we do not know much about what students learn from homework.

LaRose (2010) found homework improved students' ability to do procedural integration problems. There is evidence that students frequently complete textbook exercises by focusing on superficial features and finding procedures to mimic (Lithner, 2003). We also know that in the case of online homework, students sometimes guess answers (Dorko, 2018; in preparation; Hauk & Segalla, 2005) or sometimes type entire problems into search engines (Krause & Putnam, 2016). However, there is also evidence that students engage in mathematical sensemaking when doing online homework (Dorko, 2018; in preparation; Krause & Putnam, 2016). Homework has the potential to be a powerful learning environment and research about the nature of students' reasoning while doing homework and what they learn from different sorts of homework tasks can help instructors design homework assignments that more effectively influence students' cognitive activity.

Toward that end, this paper reports on a pilot study of student learning from homework. I sought to answer the research questions (1) *why did two calculus II instructors choose the particular problems they did and* (2) *did nine calculus II students learn what instructors intended they learn from each of fourteen problems in an online homework assignment about sequences?* While limited in scope to one assignment, the results provide initial information about what students might reasonably learn from an online homework assignment. Additionally, themes in the professors' intentions for the problem talk back to the theory by providing insight into how the professors managed the knowledge at stake across multiple milieu.

## Data Collection

The data presented here come from video recorded interviews with two calculus II professors, and video recordings and follow-up interviews with 9 calculus II students. The data were collected in the fall and spring semesters at a large public university in the U.S. Calculus II at this university is a coordinated course in which a course coordinator chooses a set of online homework problems for each section. Each professor assigns some or all of the problems the course coordinator chose. In the professor interviews, which occurred prior to the student interviews, each professor viewed the coordinator's 14 chosen problems for section 10.1, sequences. The professors had each taught the course numerous times and the problems were not new to them. For each question I asked, "would you assign this problem and why or why not?" If the professor would assign the problem I asked, "what would you hope students would learn from this problem?" I transcribed both interviews and listed what the professors hoped students would learn from each question. I then wrote questions for the student interviews based on this list, with the goal that students' verbal answers and written work would lend insight into whether the student had achieved the professors' goals for the problem. For example, in Question 1 (Figure 2), Professor B hoped students would "solidify their understanding of factorials", so I asked students "how familiar are you with factorials?". Professor A said of the sequence -1, 1, -1, 1, … with general term $\cos(n\pi)$ would

*Excerpt 1. Professor A discussing the sequence -1, 1, -1, 1 with general term* $\cos(n\pi)$

*Professor A*: [This one] is sort of a good idea because it shows that a sequence can be simpler than the way it's defined.

Hence I asked students "were you struck by the fact that the sequence -1, 1, -1, 1, … is fairly simple, but is defined by a trig function?" As another example, Professor B chose not to assign a question that asked about the convergence of $a_n = (3/8)^n$, but Professor A said he would assign this because

*Excerpt 2. Professor A discussing a question that asked about the convergence of $a_n = (3/8)^n$*

*Professor A*: I think this will naturally get them thinking more about this as a discrete set of numbers [instead of a continuous function].

Hence I asked students if they had a mental image of that sequence and if so, I asked them to describe or sketch their mental image. If students described or sketched a line, I asked "do you envision this sequence as a line or a set of points?"

Match each sequence with its general term. (Assume $n \geq 1$)

(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$ (b) -1, 1, -1, 1, … (c) 1, -1, 1, -1, … (d) $\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, \ldots$

$\cos(n\pi)$ $\frac{n!}{2^n}$ $\frac{n}{n+1}$ $\sin(n\pi)$ $(-1)^{n+1}$

*Figure 2. Question 1 in the online sequences homework.*

Each calculus II student met with me twice. In the first session, I video recorded them doing their online 10.1 homework. I did not interrupt students except to ask what they had typed on calculators. I photocopied students' written work from the session, their class notes, and any supplemental materials they viewed. In the second session, the student and I watched the video. I paused the video to ask students questions about what they did and why and to ask each question I had generated from the professors' lists of the knowledge at stake.

## Data Analysis: Why Professors Chose the Problems that they Did

I employed the constant comparative method (Strauss & Corbin, 1994) to identify themes in why the professors chose the questions they did. The professors stated their reasons for choosing the problems in terms of what they hoped students would learn from them, so the data source for this analysis was the same list generated above. I read through the list looking for similarities in the motivations the professors expressed with the items. For example, professors mentioned including problems because a particular notation or operation would be important in future topics (e.g., the notation shown in Question 4, Figure 3). Other problems they included to elicit shifts in students' cognitive activity. For example, in Question 4, Professor B said

*Excerpt 3. Professor B discussing question 4 (figure 3)*

*Professor B*: I know from experience that many of them are going to say $c_1$ is 1/5, $c_2$ is 1/8, $c_3$ is 1/11… because this notation is, is very unfamiliar to them, this idea of a sum with a variable at the end. [I hope] they would get comfortable with the idea of this kind of notation for a partial sum, and so when in the next section we start doing this all the time, they've at least done it for themselves one time.

I continued searching for similarities until I believed I had exhausted them all, then wrote an initial set of categories and their criteria. Following this, I applied the criteria to code all the items again, which allowed me to refine the criteria and to ensure each item belonged to at least one category (that is, to ensure the categories adequately described all the data). The resultant themes are presented in the next section.

Calculate the first four terms of the given sequence, starting with n = 1.

$$c_n = \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \cdots + \frac{1}{3n+2}$$

$c_1$ = ______ $c_2$ = ______ $c_3$ = _______ $c_4$ = ______

*Figure 3. Question 4 in the online sequences homework.*

## Results: Why Professors Chose the Problems that they Did

Table 1 shows the themes in why the professors selected the problems they did. These themes are not mutually exclusive, and any particular instance of student work could be coded as representing multiple themes.

Table 1. Themes in professors' reasons for assigning particular questions

| Category | Examples |
|---|---|
| Students will engage with a skill/concept specific to the content of section 10.1. | Excerpts 1, 2, 3 |
| Students will engage with a skill/concept that is important for a future topic. | Excerpt 3 |
| Students will make a connection back to a prior skill/concept (either from the current course or a past course). | Both professors hoped students would recall the use of dominant terms to find the limit of the sequence $a_n = \frac{7+n-3n^2}{7n^2+3}$ |
| Students will build number/operation sense (familiarity with numbers and operations. | The professors wanted students to gain familiarity with factorials, powers of -1, powers of 2, etc. |
| The professor would refer to the problem in class. | Excerpt 3 |
| Students will experience a cognitive shift (think about something in a different way). | Excerpt 2, 3 |

The theme 'Students will engage with a skill/concept specific to the content of section 10.1' seems obvious, but this category helped distinguish statements professors made about students connecting back to a prior skill from statements about something new in section 10.1 (e.g., notation). I discuss these findings later in the paper.

## Data Analysis: Did the Students Learn what the Professors Intended?

To analyze whether students learned what instructors intended from each question, I took each item from the previously-generated list of what professors hoped students would learn from each question and identified what would suffice as evidence that a student had met that outcome. Identifying what I would take as evidence was an iterative process in which I looked at the data from all students (all 9 students' written work and answers to the interview prompts) while I was trying to determine what would suffice as evidence. Because this was a pilot study and there is so little literature about student learning from homework, I did not know how to define "learning" for this context or what might count as evidence of it, so I was unable to establish a priori what evidence of learning might be. Looking at what students had done for each question helped me determine what it might mean for students to learn a particular part of the knowledge at stake. For example, in many of the questions (e.g., Question 4, Figure 3), the professors wanted students to gain familiarity with notation. Looking at student data for these questions helped me determine that if a student answered those questions correctly, they had made sense of the notation.

## Results: Did the Students Learn what the Professors Intended?

On the whole, the nine students achieved the goals the professors stated regarding gaining familiarity with notation, operations (e.g., factorials), number sense, vocabulary, and procedures. However, the students seldom noticed nuances the professors hoped they would notice in particular problems. I present brief examples of each below. These students' responses were representative of the entire group.

### Familiarity with notation, operations, number sense, vocabulary, and procedures

There were two problems in which professors hoped students would gain familiarity with notation. One (Question 2) was making sense of subscripts: students were given a formula for $a_n$ and directed to generate terms for $b_n = a_{n+1}$, $c_n = a_{n+3}$, and $d_n = 2a_n - a_{n+1}$. Professor A said notation "tends to trip them up", so I inferred he wanted students to become familiar with subscripts. Professor B said "I want them to be very comfortable with what $a_{n+1}$, $a_{n-1}$, what that does in the sequence." All students computed the terms correctly (some taking multiple attempts), which I took as evidence that they had made sense of the subscripts. I also asked students to describe what the subscripts meant, and they made statements such as "you would just go to like the term after… on this one you had to go to the third term after". Question 4 (Figure 3) was the other problem the professors thought was important for notation. Professor B said "I know from experience that many of them are going to say $c_1$ is 1/5, $c_2$ is 1/8, $c_3$ is 1/11… because this notation is, is very unfamiliar to them, this idea of a sum with a variable at the end. [I hope] they would get comfortable with the idea of this kind of notation for a partial sum, and so when in the next section we start doing this all the time, they've at least done it for themselves one time". Five of the students computed the terms correctly on their first try, indicating they had made sense of the notation. The other four made the error Professor B predicted, then computed the terms correctly on their second try. I took these correct computations, and students' descriptions of how they thought about the problem, as evidence that they made sense of the notation. For example, one student said at first she thought the 1/(3n+2) was "the pattern, like in the previous question" in which she had been given a general term. She described that after seeing her initial answers were wrong, "I realize[d] it was adding the terms and not just like the individual [fractions]". Another student said "I learned what to look for, and the difference between… a sequence and a sum."

Like the analysis regarding whether the students made sense of the new notation, my criterion for items the professors stated about familiarity with operations and number sense was primarily whether or not the students answered the problems correctly. For example, the professors felt questions like number 1 (Figure 2) and number 3, computing terms of $9^n/n!$, would help students "recogniz[e] you know factorials, powers of 2, changes of sign… they're getting to practice that" and "they're getting to use powers, maybe a power they're not familiar with… so they'll see how those numbers come out". I took correct answers to problem 1 (Figure 2) as evidence students gained familiarity with factorials, powers of 2, and changes of sign because students either seemed to do this in their heads (writing nothing or typing nothing into a calculator) or took the general terms and wrote the first several terms of the sequences before matching the answers. That is, students appeared to engage in the computations with the general terms in question 1 and this engagement represented their gaining more familiarity with particular powers and operations. In question 3, computing terms of $c_n = 9^n/n!$, six students either did the factorial calculations in their heads and/or wrote the factorials as products (e.g., $(9^4)/(1*2*3*4)$). I took this as evidence that they gained familiarity with the factorial. Three students typed the $9^n/n!$ into their calculators. The calculators outputted simplified answers, which did not give students an opportunity to see powers of 9 or how the factorial affected the terms.

Five questions with various sequences were stated like Question 9 (Figure 4) in which students selected a multiple choice option that Professor A considered "checking vocabulary." Though students did not answer all these questions correctly, with one exception, they always checked 'converges' if they inputted a limit that was a real number and 'diverges' if they inputted $\infty$ or $-\infty$. Two students looked at the definition of converge in the textbook or their notes. I took this and the internal consistency of all students' answers to the two parts of the problem as evidence that they either knew what 'convergence' and 'divergence' meant before starting the assignment, or (in the case of the two students who looked up the definitions) they learned it while doing the assignment.

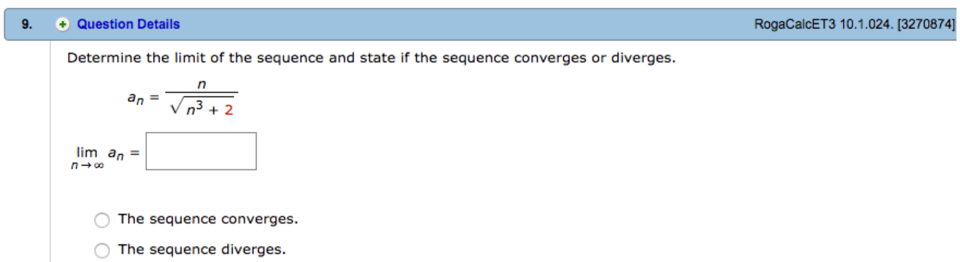


*Figure 4. Question 9 in the online sequences homework.*

In questions like question 1 (Figure 2), questions 2 and 3 (described above), and question 4 (Figure 3), the professors described they wanted students to become familiar with the procedure of generating terms of a sequence. All of the students answered these questions correctly, which served as partial evidence that they had gained familiarity with the procedures in each case. In summary, on the whole the nine students achieved the goals the professors stated regarding gaining familiarity with notation, operations (e.g., factorials), number sense, and vocabulary.

These results support the efficacy of an online homework program with multiple attempts per question for helping students make sense of new notations, gain familiarity with operations and powers of numbers, learn the meanings of new vocabulary, and practice procedures. An important caveat is that if the goal is for students to gain familiarity with operations (e.g., factorials), it may be best to encourage students to write computations by hand instead of relying on a calculator.

**Nuances**

While the homework problems supported students in learning or practicing notation, procedures, and operations, students largely missed the nuances the professors hoped students would notice in the problems. This may be because the professors' goals for the problems asked for something that was not a necessary conception for the students to have in order to get the correct answer, and not a connection that was directly asked of the students. For example, no student picked up on Professor A's desired take-away for $\cos(n\pi)$ and the sequence -1, 1, -1, 1, … (Excerpt 1). Calvin, one of the students, said

*Excerpt 4. Calvin discussing* $\cos(n\pi)$ *and the sequence -1, 1, -1, 1*

*Interviewer*: Did it surprise you… so we have a sequence that's fairly simple, right? Because it's -1, 1, -1, 1 but it's defined with a trig function.

*Calvin*: I mean not really. I mean back when I was first learning trig stuff, like the emphasis on the graph and how it was alternating… I didn't really think about it.

Similarly, students did not imagine $a_n = (3/8)^n$ as a discrete set of points, as Professor A intended (Excerpt 2). The students who described mental images of this sequence described or drew continuous functions.

In the next section, I discuss the results and make connections between the themes in why the professors chose the problems and whether students learned what the professors intended.

## Discussion and Conclusion

These results support others' findings that online homework can improve students' fluency with procedures and notation (LaRose, 2010). However, students missed some of the nuances the professors hoped they would take away from the problems. There were two problems professors hoped would cause cognitive shifts for students, but only one problem was successful in doing so. Professors chose problems that helped students recall prior learning and connect it to the new content, problems for students to practice content particular to sequences and their computation, problems that would provide a foundation for future content, problems they wanted to talk about in class, and problems that they hoped would cause students to think about something differently. This list could be informative for new instructors in thinking about what to include in a homework assignment.

White and Mesa (2014) found variation in the cognitive orientation of tasks across milieu (homework, worksheets, exams) and instructors. The professors in this study selected problems that were largely procedural, expressing they wanted students to be exposed to these problems beforehand so they could discuss the details in class. For example, in a problem that directed students to use limit laws and theorems to find the limit of $a_n = 9^n/n!$, Professor B said he did the example $11^n/n!$ in class …

*Excerpt 5. Professor B discussing* $a_n = 9^n/n!$

*Professor B*: ... so it, it went up for a bit longer before it started to come down… they either didn't know the limit or they thought the limit was infinity. And then you know I talked through factoring everything and realizing that after we get to this peak, things start to come down, and they start to come down kind of fast because we're multiplying by these numbers that are less than 1 all the time….We have to actually manipulate the factorial as a product now to, to see the answer.

Similarly, the professors wanted the students to have experienced computing partial sums so they were familiar with it for the lesson on series (Excerpt 3). In summary, the findings suggest the professors made intentional decisions about managing the knowledge at stake across milieu.

These results have many implications for future research. One avenue would be investigating what students learn from homework problems that are more conceptual in nature. In particular, online homework platforms have the advantage of allowing students multiple attempts and providing immediate feedback, and research should examine how we can leverage these systems to influence the cognitive bases of students' activity.

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