Factors Influencing Linear Algebra Instructors' Decision to Implement Inquiry-Oriented Instruction

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This study investigates factors that influence instructors' decisions to implement inquiry-oriented instruction. We analyzed entrance interviews with twelve Linear Algebra instructors, who participated in the Teaching Inquiry-Oriented Mathematics: Establishing Supports professional development project, to better understand the reasons why the instructors chose to shift from traditional lecturing to inquiry-oriented instructional approaches. We found three internal and three external factors that influenced the participating instructors' choice to teach the inquiry-oriented Linear Algebra course. Implications for future research are discussed.

Keywords: inquiry-oriented instruction, linear algebra

Student-centered instructional approaches have received significant attention over the last several years. Although lecture is still the predominant way of teaching undergraduate mathematics courses (Eagan, 2016; Johnson, Keller, & Fukawa-Connelly, 2017), researchers suggest that implementing active student-centered instructional approaches, such as Inquiry-Based Learning (IBL) or Inquiry-Oriented Instruction (IOI), may be more beneficial for students' achievement, affect, and persistence in undergraduate mathematics (e.g., Freeman, Eddy, McDonough, Smith, Okoroafor, Jordt, & Wenderoth, 2014; Laursen, Hassi, Kogan, & Weston, 2014). To support the claim that inquiry-based teaching promotes positive student learning outcomes, Freeman et al. (2014) conducted a meta-analysis of 225 studies that compared achievement outcomes of students in undergraduate STEM courses taught via either active learning or traditional lecture approaches. The meta-analysis concluded that using teaching approaches that gave students opportunities to actively participate, rather than passively listen, reduced student failure rates and raised students' scores on exams. Research has highlighted the effectiveness of IBL and IOI, so there is a need for training mathematics instructors to adopt such instructional approaches. Several projects have been designed to train and support instructors in student-centered teaching approaches, such as NExT Project, the Academy of Inquiry-Based Learning, and the TIMES (Teaching Inquiry-Oriented Mathematics: Establishing Support) project, which designed inquiry-based curriculum for undergraduate mathematics courses and provided professional development for instructors to implement student-centered instructional approaches. This study explores what influences instructors' decisions to pursue such professional development opportunities to learn to implement IOI.

We specifically focus on Linear Algebra instructors. Knowledge of Linear Algebra is vital in multiple areas of science. In many universities, the course of Linear Algebra is usually taken by students of diverse backgrounds and educational pursuits. Instructors' pedagogical approaches play a crucial role in influencing students' interest, motivation, and success in this course. Therefore, it is worthwhile to explore reasons why instructors choose to use a certain instructional approach to teach Linear Algebra. The purpose of the present study is to explore common factors that motivate instructors to use IOI to teach Linear Algebra. The following research question was addressed: What factors influence Linear Algebra instructors' decision to implement IOI?

Literature Review and Theoretical Perspective

Student-centered teaching approaches differ from what is considered traditional lecturing in mathematics courses. The aim of student-centered instructional approaches is to enhance students' problem-solving skills, giving them opportunities to generate ideas, ask their own questions, and develop strategies for answering them (Laursen, Hassi, Kogan, & Weston, 2014). In inquiry-oriented classrooms, students are actively engaged in producing their own mathematical ideas in solving problems, rather than repeating algorithms demonstrated by the teacher. Students present their solutions in front of the whole class or in small groups, while other students critically analyze their peer's solution and provide their feedback. IBL and IOI give students opportunities to "do mathematics like research mathematicians do mathematics" (Yoshinobu & Jones, 2012, p. 307). A growing body of research studies suggests student-centered teaching has positive effects on student learning in undergraduate mathematics (e.g., Freeman, Eddy, McDonough, Smith, Okoroafor, Jordt, & Wenderoth, 2014; Laursen et al., 2014). For instance, Kogan and Laursen (2014) analyzed data from 100 sections of 40 courses and found that students who had been engaged in an inquiry-oriented classroom were more likely to succeed in subsequent mathematics courses than students who had been taught via the traditional lecture approach. This highlights the benefits associated with incorporating student-centered instruction.

A historical predecessor of IBL is the Moore Method, named after mathematician R. L. Moore (Coppin, Mahavier, & May, 2009). The implementation of this method varies among instructors, but the core idea is that instead of using a certain textbook, the students are given a list of theorems, which they are expected to prove using given definitions. After the students prove a theorem, they present it in class, while their class peers evaluate the validity of the proof. There are several differences between the Moore Method and IOI, albeit they are both forms of student-centered instruction; an example of such would be that students' collaboration is prohibited in the Moore Method, whereas student collaboration is expected in IOI.

IOI is informed by Realistic Mathematics Education, an instructional design theory, which considers mathematics as a human activity (Freudhenthal, 1973). One of the most important heuristics of RME is that the instruction should provide students opportunities to reinvent key mathematical concepts with the guidance of the instructor (Stephan, Underwood-Gregg, & Yackel). In this guided reinvention process, mathematical concepts are not presented to students by the instructor, as in traditional lecture. In contrast, the instructor poses carefully designed mathematics tasks for the students to collaboratively work on. These tasks are designed to promote the emergence of the mathematical concepts, as students develop an intuitive understanding of the concepts. The instructor then formalizes the students' knowledge of the mathematical concepts. In short, guided reinvention in IOI involves the students reinventing mathematical concepts with the support of the instructor.

The instructor plays an essential role in IOI courses (Rasmussen & Kwon, 2007). Kuster, Johnson, Keene, and Andrews-Larson (2017) emphasized, "by inquiring into student thinking, teachers are able to support students in generating more sophisticated ways of reasoning" (p. 6). Cobb, Wood, and Yackel (1993) argued that the teacher plays an important role in developing students' conceptual knowledge and providing opportunities to share the acquired knowledge with peers through collective discussion. Along with asking students questions and facilitating discussions, instructors are responsible for establishing and sustaining classroom norms which allow students to share with their mathematical ideas (Stephan et al., 2014).

Several studies over the past decade have examined factors that influence instructors' decisions to move away from using traditional lecture to implement inquiry-based teaching approaches. Johnson, Keller, and Fukawa-Connelly (2017a) investigated what affordances and constraints on the use of non-lecture practices Abstract Algebra "lecturers" perceive. The authors administered a national survey to Abstract Algebra instructors, which gathered data on their typical teaching practices, beliefs about teaching and learning, and contextual affordances and constraints for using certain teaching practices. The data revealed a number of contradictions in the participants' responses. On one hand, several instructors suggested a lack of time, curricular resources, knowledge, and supports were reasons why they would not choose to use instructional methods other than lecturing. On the other hand, the same instructors claimed that they might have time for redesigning their instruction, they did not feel pressure from their departments to cover a certain amount of material, and there were funds available for teaching professional development opportunities. Despite this reluctance to adopt student-centered instructional practices, Johnson et al. found that 65% of lecturers from institutions that offer Bachelor's and Master's degrees and 48% of lecturers from PhD-granting institutions would consider switching to non-lecturing instructional approaches. In another study, Johnson, Keller, Peterson, and Fukawa-Connelly (2017b) explored Abstract Algebra teachers' beliefs, habits, and constraints at Bachelors-granting institutions, i.e. traditionally "teaching colleges." Johnson et al. (2017b) investigated the extent to which these Abstract Algebra instructors employed non-lecture approaches. They found that in these institutions, lecturing is the predominant way of teaching. The authors concluded that reformers still have a long way to go in helping instructors implement student-centered practices in mathematics. This motivated the present study to explore why instructors chose to switch to using a non-lecture approach.

We follow Henderson and Dancy's (2009) theoretical framework of aspects that influence instructional practices: experience with and attitudes toward teaching innovations, instructional goals, and perception of department support. We expand this theoretical framework by adding instructors' beliefs about students' difficulties in learning Linear Algebra. We also conduct more detailed analysis of external pressures that affect instructors' choice to use IOI. One of the goals of our research is to further explore Johnson et al.'s (2017a) findings regarding the influence of departmental pressure on the instructors' choice to use certain pedagogical practices. We also aim to discover other influential factors that were not previously found in the literature.

Methods

The following section describes the context of the study, the teaching experience of the participating instructors, and the methods we used for data collection and analysis.

Context of the Study

This study is part of a larger research program, the NSF-funded TIMES project, which is a professional development program designed to support undergraduate mathematics instructors of Linear Algebra, Differential Equations, and Abstract Algebra in learning how to implement IOI. The professional development program provided instructors with training in a three-day summer workshop, as well as support through the provision of curriculum materials and weekly online peer working groups. This study explores the factors that influenced Linear Algebra instructors to implement IOI through participating in the TIMES project.

Participants

Thirty-six undergraduate mathematics instructors participated in the TIMES project as fellows. This study considers a subset of twelve of those instructors, all of whom taught the Inquiry-Oriented Linear Algebra (IOLA) course. These instructors came from a variety of institutions across the United States. The participating instructors exhibited differences in their amount of experience in teaching Linear Algebra. Two (17%) of the instructors had taught Linear Algebra three or more times before, six (50%) of the instructors taught this course a couple of times, and four (33%) of the instructors of instructors had never taught the course prior to teaching the IOLA course. The instructors also exhibited differences in their previously used teaching practices (i.e. lecture, IBL, or a combination of both). Five (42%) of the instructors described their own teaching practice as mostly lecture, five (42%) of the instructors claimed they used mostly IBL methods, and two (16%) of the instructors claimed to use both methods.

Data Collection

Semi-structured interviews were conducted by project personnel with each of the instructors after they took part in summer workshops, in which they learned how to implement the IOLA curriculum. These interviews took place before the teachers began using IOI in the classroom. An interview protocol was written and administered in each interview to ensure the participants responded to the same questions. Some follow up questions were posed by the interviewer to elicit clarification or more detailed responses from participants. The questions prompted the instructors to describe their past teaching experiences, their reasons for wanting to implement IOI, and the nature of the support they received from their colleagues. The interviews were audio recorded and transcribed for retrospective analysis.

Data Analysis

The first author analyzed the interview transcripts using thematic analysis, coding common themes that emerged from the data (Roulston, 2010). Initial codes were produced based on the author's interpretation of the data. Similar codes were reorganized into categories during second cycle coding (Miles, Huberman, & Saldana, 2014). To ensure dependability of the qualitative analysis, both authors met to discuss the codes and their pertinence to answering the research question. The authors compared the different instructors' responses to find trends in the topics the participants discussed.

Results

As themes emerged from the data, we recognized the themes could be categorized as either internal factors related to instructors' interests, beliefs, and goals, or external factors related to departmental or student expectations. Our research findings are presented in the following two sections organized by the nature of factors that influence the instructors' choice to use IOI. The first section discusses the internal factors, and the second section describes the external factors that appeared to influence the instructors' choice to implement IOI. The discussion in each of the subsequent sections describes the nature of the different internal and external factors that were evident in the interview data.

Internal factors

In this section we describe internal factors that seemed to influence the instructors' decision to implement IOI. These internal factors include the instructors' interests in implementing IOI, beliefs about students' difficulty in learning concepts in Linear Algebra, and instructional goals.

Instructors' interests in IOI. The instructors' interests in IOI seemed to influence their choice to pursue inquiry-oriented instructional methods, so we explored the instructors' given reasons for their interests in IOI. Several instructors (42%) cited their past student experience as a reason to move away from traditional lecturing. Some of the representatives of this group had negative experiences as students of lecturers. One such instructor claimed, "Reflecting back on my own schooling, I fell asleep in Calculus and in most of my math classes because I only had experienced a lecture style." Other instructors experienced positive effects on their learning after participating in inquiry-based courses. One of these instructors asserted, "When I was an undergraduate student, I had IBL topology. I hated it when taking it, but it helped me greatly." These past student experiences were influential in motivating these instructors to develop interests in IOI.

Many instructors (25%) attributed their interests in IOI to their involvement in professional development events, such as conferences, professional seminars, and workshops. We also found 15% of the respondents mentioned they were satisfied by their lecture-based approach, but they were curious if there were other ways of teaching that could be more beneficial for students. Another 15% of the instructors were inspired by the successful inquiry-oriented practices of their colleagues and the desire of their department heads to incorporate innovative ways of teaching in mathematics courses. All of these reasons for instructors' interests in implementing IOI seemed to influence the instructors' decisions to change their instructional approaches.

Instructors' beliefs about students' difficulty in learning Linear Algebra. The instructors' perceptions of students' difficulty in learning Linear Algebra influenced their decision to use IOI in the classroom. When asked what they perceived as the most difficult aspect of the course for students, half of the instructors referenced the shift from doing basic computations to solving abstract problems. One of these instructors claimed the most difficult part of Linear Algebra is, "the abstract nature of the subject, especially for those who have just been through the calculus series." Additionally, 25% of the instructors cited formal proof writing as the most challenging part of the course, and 17% of the respondents argued that the greatest difficulty the students faced in Linear Algebra was understanding how "everything is interconnected." None of the instructors cited computation as a challenging aspect of the course.

These beliefs about what students struggle with in the Linear Algebra course contributed to the instructors' decision to implement IOI. Several instructors highlighted the usefulness of inquiry-oriented teaching in helping students gain deep understanding of abstract concepts. For instance, one instructor said, "I discovered that students have a quite difficult time when starting the concept of basis and span… so I started thinking this is where the IBL can be useful." The instructors perceived IOI as a method that could facilitate students' development of meaningful understanding of the abstract concepts in the course. Overall, instructors' beliefs regarding students' difficulty in learning Linear Algebra and their views regarding the potential benefits of IOI in helping students overcome these difficulties served as contributing factors that influenced the instructors' decision to implement IOI.

Instructional goals. The instructors' teaching goals seemed to influence the instructors' decision to implement IOI. The instructors seemed to believe implementing IOI would provide a way for them to achieve their instructional goals. Half of the instructors had instructional goals of helping their students be able to "build arguments," "explain their reasoning", "reflect on others ideas," and "provide critical feedback." One such instructor described how using IOI could help in pursuing these goals, claiming, "If [the students] have conversations with others early, later they can have conversations with themselves." This instructor seemed to believe IOI

provided opportunities for students to build habits of communicating their mathematical reasoning, which would be useful for the students as they take future mathematics courses. This notion was echoed in the instructional goal held by 33% of the instructors, which was that of fostering greater mathematical maturity in their students and preparing them for other mathematics courses. These instructors viewed IOI as a way to achieve their instructional goals of giving students opportunities to communicate about mathematics and develop mathematical maturity. Therefore, the instructors' instructional goals were influential in their decision to implement IOI.

External factors

The following section addresses the external factors that seemed to influence the instructors' decision to employ inquiry-oriented teaching methods. These external factors include pressure from student evaluations, departmental support, and content coverage expectations.

Pressure from the effect of student evaluations on tenure status. The instructors' perceived pressure from the effect of student evaluations on their tenure status might have a negative influence on some instructors' decision to implement IOI. One instructor described waiting to try innovative teaching methods until he was tenured because he was cautioned "not to rock the boat with students until after [his] job is secured." Students' potential lack of appreciation for innovative teaching may be exhibited in poor student evaluations of instruction, which could have a negative impact on instructors' tenure process. Another instructor confessed that his department chair advised him not to try anything new until he had been tenured, since negative evaluations may adversely affect his pursuit of tenure. This fear of poor student evaluations might deter some instructors from choosing to implement IOI. However, the majority of the instructors (58%) participating in this professional development were untenured or were not on a tenure track, so this worry of negative student evaluations did not seem to deter them from choosing to implement IOI. Some of these instructors mentioned that they did not really worry about students' evaluations. Overall, some instructors felt pressure of potentially being negatively evaluated by students, but this did not deter them from choosing to implement IOI. However, fear of negative student evaluations can influence instructors to not choose to incorporate innovative teaching approaches.

Departmental support. Several instructors perceived supportive attitudes from their department chairs and colleagues regarding their intent to implement innovative teaching methods. One of the instructors specified the nature of this support from his department chair, claiming, for "anybody who goes in with a new idea, and whether it is about an instructional approach or instruction needs for the classroom, he is supportive in finding a way to make those things happen." The instructors claimed to be given full autonomy to implement whatever teaching methods they chose. The instructors asserted that, in general, most of their colleagues were very supportive of their decision to implement inquiry-oriented approach. This supportive departmental environment seemed to influence instructors' decisions to implement IOI, in that they did not feel any discouragement from colleagues that would inhibit them from doing so.

Content coverage expectations. The instructors generally felt no constraint to comply with covering specific topics other than those usually covered in Linear Algebra. One instructor specified that the only concepts he was required to cover were vector spaces, maps, eigenvalues, eigenvectors, and some proofs. Another instructor claimed, "I don't have to serve anybody else's wishes." Several instructors reported they were not required to cover a certain list of topics, assign certain homework assignments, or administer certain exams. One of the instructors

mentioned that there was a textbook he was required to use, but he was encouraged to incorporate supplemental instructional material. The instructors generally did not feel much pressure from the department to cover certain content. This lack of curriculum constraints and freedom to use alternative curriculum materials seemed to serve as contributing factors in the instructors' decision to implement IOI.

Discussion

This study explored influential factors that seemed to affect instructors' decisions to implement IOI. We found three internal and three external factors that seemed to influence the participating instructors' choice to teach the IOLA course. The internal factors included instructors' interests in IOI, beliefs about students' difficulty in learning Linear Algebra, and instructional goals. With this finding, we propose expanding Henderson and Dancy's (2009) theoretical framework of aspects that characterize instructional practices by adding the instructors' beliefs about students' difficulties in learning mathematical concepts.

The instructors in this study viewed implementing IOI as a way to achieve their instructional goals. Future research can explore how professional development opportunities can leverage instructors' beliefs and goals to align them with the aims of the professional development program. Furthermore, these instructors had interests in the potential benefits of IOI, which influenced their decision to change their instructional approach to IOI. Therefore, dissemination efforts need to be made to increase mathematics instructors' awareness of the benefits of IOI and spark instructors' interests in using non-lecture teaching approaches.

The external factors that influenced instructors' decisions to incorporate IOI into their teaching include pressure from student evaluations, departmental support, and content coverage expectations. Typically, mathematics instructors' arguments against using non-lecture approaches like IOI reference departmental constraints and coverage concerns (Johnson et al., 2017a). Some instructors believe using primarily lecture-based instructional approaches helps them cover all the course content within certain time constraints, and they believe using non-lecture approaches would not allow them to do so. Departmental requirements and lack of support from colleagues can also deter instructors from implementing IOI. Contradictory to these typical excuses for not using innovative instructional methods, the instructors in this study generally did not receive discouragement from other faculty members for implementing IOI, nor did they reveal any pressure from requirements to cover a specific amount of content. The instructors generally felt a sense of support from their department chairs for choosing to use IOI in their Linear Algebra course. This finding could imply that instructors with supportive department chairs are more willing to try using IOI, or this could mean that arguments against implementing IOI concerning coverage constraints and departmental discouragement are ill-posed. Further research is needed to explore these hypotheses. There is also a need to investigate the source of mathematics instructors' perceived pressures to cover certain content and comply with supposed departmental expectations. Further research can explore how to help mathematics instructors, particularly lecturers, see the potential benefits of using IOI in their classrooms to allow for more widespread adoption of IOI in undergraduate mathematics courses.

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