Examining Questions as Written Feedback in Undergraduate Proof-Writing Mathematics Courses

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Abstract

The practice of providing written feedback on an undergraduate student's proof in the form of asking questions is striking in that professors do not know whether the student attempts to answer the questions. This phenomenon leads us to investigate the reasons why professors ask questions as written feedback. We analyze the written questions of four professors teaching abstract algebra and real analysis at a medium-sized, rural, comprehensive public university in the northeast. We find that these four professors most frequently ask questions that either seek further explanation from students or address a mathematical detail within their proof. In some cases, the professors answer the questions they ask as written feedback. Overall, the professors ask questions as written feedback to encourage students' thinking, thereby engaging students in the proof-writing process and improving the students' proof production skills.

Keywords: Feedback on Proof, Questioning, Proof Instruction, Written Feedback

Introduction

Professors who teach undergraduate courses often give students written feedback on their writing and assessments, but some feedback results in miscommunication between the professor and student (Price, Handley, Millar & Donovan, 2010). Many students find written feedback unhelpful or are uncertain of how to use to it (Amrhein & Nassaji, 2010; Price, et al., 2010; Vardi, 2009). In proof-writing mathematics classes, students misinterpreted the intention of written feedback (Byrne, Hanusch, Moore, Fukawa-Connelly, 2017). As professors report spending considerable time writing feedback on student proofs (Moore, 2016), it is desired that the feedback will maximize student understanding. The first step in evaluating the effectiveness of feedback is to research the current written feedback practices of collegiate mathematics professors, which responds to the call from Speer, Smith & Horvath (2010) for additional research on collegiate mathematics instruction.

We defined written feedback as the language and annotations that a professor leaves on a student's written work. Two types of written feedback are excluded from our analysis: check marks and numeric scores. Check marks are excluded because they convey little information, except that the professor read and accepted that component of the proof. Although feedback is often provided to justify numerical scores (Glover & Brown, 2006; Price et al., 2010), we disregard numerical scores. This decision is for privacy concerns and FERPA regulations, but also because inconsistencies were found between the feedback professors provided and the scores they assigned, as professors assigned significantly different scores to the same proof (Lew & Mejía-Ramos, in press; Moore, 2016; Miller, Infante, Weber, 2017).

This paper is part of a larger project examining all written feedback on proofs, and this paper examined the phenomenon of a professor asking a question as written feedback. This subset of the feedback had an inherent contradiction, as the question should be answered, yet none of the professors required the students to formally answer these questions. An example of this phenomenon is shown in Figure 1, where the student introduced a new variable on an abstract algebra homework assignment but failed to describe the properties of this variable. The professor chose to use the question "What is x?" to provide feedback.

Furthermore, professors occasionally answer their own questions in writing on their student's paper. Sometimes the answer is provided as a statement, but in other cases, such as in Figure 1, the answer is provided in a more detailed follow up question.

In this article, we addressed the following research questions:

1. What types of questions do professors write on student proofs?
2. Why do professors choose to ask questions as a form of written feedback?
3. Why do professors choose to write answers for some of the questions they ask?

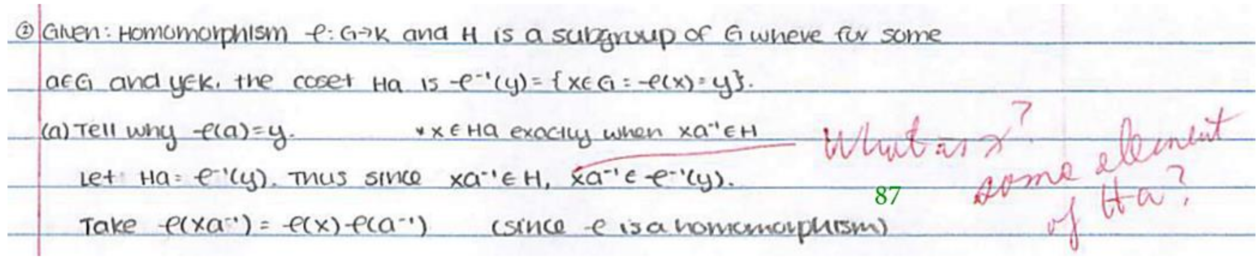

Figure 1 An instance of using a question as written feedback.

Literature Review

Theoretical perspective of feedback

Evans (2013) proposed a constructivist model which views feedback as an exchange between the instructor and student, termed the feedback landscape. Within this landscape, all instructors and students interpret the feedback and work through a personal buffer zone which is informed by the individual's social and cognitive factors. Since these buffers are individualized, it is within these zones that the intended meaning of the feedback can be lost.

Theoretical perspective of questions in instruction

Rowe (1986) described classroom interactions as a game with two players: the teacher and the set of students. This game has four moves:

1. Structuring: giving directions, stating procedures, suggesting changes.
2. Soliciting: asking questions.
3. Responding: answering solicitations, expanding on a structuring move, reporting data, or continuing a line of reasoning.
4. Reacting: evaluating statements made by self or other player (Rowe, 1986, p. 46).

These moves can be initiated by any player and satisfaction is highest when each player utilizes all four moves. Rowe argues that when the teacher increases the wait time between moves 2 and 3 and between moves 3 and 4, then the students complete moves 1, 2 and 4 more frequently.

For this study, we viewed these four steps as a complete questioning sequence, no matter who completed each move. We focus on written assessments (homework, quizzes and exams) where the professor chooses the proof tasks, the student responds in writing, and then the professor reacts through the written feedback. As such, the professor always completes moves 1, 2 and 4, and the student always completes move 3. However, during move 4, the professor sometimes initiated a new questioning sequence by asking a question within their feedback. These sequences are usually incomplete, as moves 3 and 4 may not be completed.

Written feedback on writing

Several studies investigated written feedback on writing in undergraduate courses. University science students found written feedback on their work useful, especially feedback that helped them to understand where they had gone wrong (Brown and Glover, 2006). Walker (2009) expressed that written comments should be classified as feedback only if it is "usable" or can be implemented by students. Unfortunately, many studies concluded that a high proportion of comments are considered unusable to students and recognized the need for an improvement in the practice of commenting on written assignments (Amrhein & Nassaji, 2010; Mulliner & Tucker, 2017; Vardi, 2009; Walker, 2009). The present study took a step toward improving feedback in the context of proof writing in undergraduate mathematics courses, although we do not investigate the student perspective directly at this time.

Written feedback on proofs

While several studies examined written feedback on writing assignments, only a few focused explicitly on feedback on mathematical proofs. Professors valued several characteristics when evaluating student proofs, including logical validity, clarity of writing and demonstration of understanding (Moore, 2016). Linguistic and notational conventions are also valued by the professors, such as using $\emptyset$ to mean "the empty set" rather than just the word "empty" (Lew & Mejia-Ramos, in press; Moore, 2016). Scoring varied greatly between professors, with ranges up to 48% observed on the same proof (Miller, Infante & Weber, 2018; Moore, 2016). The professors assigned scores based on their perceptions of student thinking, the severity of the error, and whether the proof was written in a timed or untimed setting (Lew & Mejia-Ramos, in press; Miller, Infante & Weber, 2018, Moore, 2016). These three studies established that grading and providing feedback are complex practices with competing priorities and beliefs.

Only one study, Byrne, et al. (2018), investigated feedback from the student's perspective. In this study, the undergraduate students interpreted the written feedback on sample proofs, and then rewrote each proof incorporating the feedback. The students usually addressed the feedback in the rewrite, even when they could not express the rationale for the comment. Furthermore, the students attributed much of the feedback to linguistic conventions in mathematics, even when the feedback addressed the logical validity of the proof.

All four of these studies utilized clinical interviews, and none occurred in a classroom setting. The clinical setting removed genuine communication from the feedback process and restricted the opportunities to observe the buffer zones of the faculty and students. Additionally, the previous studies focused on proofs at the transition-to-proof level. This study, in contrast, used the feedback given by professors to their own students and more accurately reflected genuine instructional practice. The mathematical content of real analysis and abstract algebra added an additional layer of complexity that allowed us to see how feedback on the mathematical content interplayed with feedback on general proof techniques and proof writing.

Questioning in class

While there is significant research on the value of written feedback, no research focused on the specific phenomenon of providing feedback in the form of a question. On the other hand, many researchers found oral questioning to be valuable in the K-12 classroom, especially for probing student thinking and promoting higher-order thinking (Acar & Kilic, 2011; Almeida, 2010; Burns, 1985; Martino & Maher, 1999). We note that Speer et al. (2010) established that instructor questioning practices have not yet been researched within collegiate mathematics classrooms.

Methods

Subjects and data sources

The subjects in the study were two instructors of abstract algebra and two instructors of real analysis at a comprehensive undergraduate institution, with one section of each course offered in a fall semester and one section of each in a spring semester. Each professor had taught the course multiple times previously and held the rank of associate professor or professor. The professors maintained full control over their course during the study, including the textbook, the nature and frequency of assignments and assessments, and how they chose to give feedback to students. The graded papers were scanned before being returned to students. The papers were then redacted to remove all identifying information about the institution, the professor, the students, and to remove grade information. Table 1 shows the number of student participants, the number of homework assignments, quizzes, and exams that were collected in the course, and the total number of questions asked in writing by each professor.

Table 1 A summary of the participants and the items considered in the study.

Course	**Fall Algebra**	**Fall Analysis**	**Spring Algebra**	**Spring Analysis**
Professor	A	B	C	D
No. of students	5	10	15	8
No. of HW/Quiz/Exam	5/6/2	9/10/2	24/3/4	24/0/2
No. of questions asked	59	134	128	247

Analysis technique

After redacting each document and numbering the feedback, we assembled a spreadsheet containing the text of every question. We did not include question marks with no text, because a question mark conveyed significantly less information to the students than a question with text.

We utilized the constant comparative method (cf. Creswell, 2013), to sort the questions into clusters. Eventually we established five clusters: *drawing attention to detail*, *seeking further explanation, questioning assumptions, expressing confusion*, and *addressing proof structure*. We note that the descriptions of the clusters are not mutually exclusive, and we made a judgment call regarding which description seems most reasonable when more than one cluster applies.

After our initial coding, we interviewed all four professors asking them to describe why they chose to write questions as feedback in general. Then, we asked each professor to review a purposeful sample of 12 items of feedback, and to provide an explanation as to why they chose to use a question for the feedback. We used the interviews to triangulate our coding and found that the professors’ descriptions aligned with our coding in all but five cases. In each case, we originally considered multiple clusters, but the professor emphasized a different cluster than the one we assigned. In such cases, we changed the cluster to match the professor’s description.

Results

Types of questions

The analysis process resulted in five clusters for classifying written questions*: drawing attention to details, seeking further explanation, questioning assumptions, expressing confusion*, and *addressing proof structure*. We defined *drawing attention to detail* as questions that ask if the details provided by the student are sufficient. Within this category, there were two subcategories: mathematically focused and language focused. Mathematically focused questions address the student's use of notation or computational work, whereas language focused questions pertain to specific word choice and phrasing made by the students.

Since justification is a cornerstone of proof, many questions *sought additional explanations* from the students. A common example of this type of question was "why is that true?" or "how do you know…", and an indication as to which line needs the explanation. The cluster, *questioning assumptions*, concerned questions that point out false assumptions made by the student. This cluster differed from *drawing attention to detail* in that *questioning assumptions* pointed out false assumptions that were made or cases that were forgotten. Some *seeking explanation* questions appear to question assumptions, but in those instances the assumptions made are typically true and simply require further explanation.

The *expressing confusion* cluster contained questions where the professor indicated confusion about the student's writing. Common questions included "what does this mean?" and "huh?" The professors also used question marks to indicate confusion such as Professor B who claimed, "But a question mark by itself really means, 'This doesn't make any sense.'"

Finally, questions in the *addressing proof structure* cluster addressed the choices made by the student regarding the type of proof or the completeness of the proof. These questions focused on the framework of the proof, instead of being detail or explanation focused.

Each professor asked questions in each cluster, except professor A. Across the four classes, seeking explanation and mathematically focused questions were the most prevalent. These findings were unsurprising as we expected students to support their claims in proof-writing classes and to occasionally make errors using new notations and concepts.

Table 2 A summary of the types of questions asked as written feedback for each professor.

	Fall Algebra n=59		**Spr. Algebra** n=129		**Fall Analysis** n=134		**Spr. Analysis** n=247		**Overall** n=569	
Detail-Math	n= 15	25.4%	n=33	25.6%	n= 45	33.6%	n=100	40.5%	n=193	33.9%
Detail-Lang	n= 3	5.1%	n=12	9.3%	n=12	9.0%	n=5	2.0%	n=32	5.6%
Detail-Other	n=0	0%	n=0	0%	n=0	0%	n=2	0.8%	n=2	0.4%
Explanation	n= 29	49.2%	n=66	51.2%	n=37	27.6%	n=99	40.1%	n=231	40.6%
Assumptions	n=3	5.1%	n=3	2.3%	n=33	24.6%	n=21	8.5%	n=60	10.5%
Proof Struct.	n=0	0%	n=8	6.2%	n=1	0.7%	n=14	5.7%	n=23	4.0%
Confusion	n=9	15.3%	n=7	5.4%	n=6	4.5%	n=6	2.4%	n=28	4.9%

Although the sample is too small to support generalization, the data suggested that course subject may impact the types of questions asked by the professors. In both algebra classes, roughly half of the questions sought explanations from students, and drawing attention to the mathematical details accounted for another quarter of the questions. In contrast, the seeking explanation cluster was a smaller portion of the questions asked in the analysis courses and drawing attention to mathematical detailed was a larger portion of the questions asked.

Why professors ask questions as feedback

While all the professors asked their students questions, they have not examined their reasons for asking a question as opposed to another form of written feedback. Professor D explained, "I don't know if it's a thoughtful, considered decision. 'Let's see, should I ask a question, or should I write a declarative statement?' I don't know if I'm thinking about it that carefully." When asked about his decision to pose a question, Professor C repeatedly responded, "I think that's just a personal style."

Professors also asked questions to alert students that their work is unsatisfactory in a "non-insulting way." Professor C explained, "It's also the same as telling them that I think something is inadequate without saying it that way." Similarly, Professor A said she sometimes avoids explicitly telling her students they are wrong, preferring to pose a question. Therefore, questions are perceived by the professor as a less harsh method of critiquing students' work.

Collectively, the professors stated that they asked their students questions, so the students will reflect upon their work. Professor D claimed he asks his students questions to

> … guide them to ask the right question to kind of correct their mistake by just knowing which question they should be asking themselves. Because, I guess as I think about it, having been a math student myself for many years, sometimes if you know the right question to ask yourself, you're well on the way to answering the question correctly.

The professors used questions as a mechanism to guide students to improved self-reflection, with the aim of improving future proof-production.

Asking questions as written feedback gives professors the opportunity to stimulate students' thought process and lead students to correct solutions. Professor D attempted to "redirect [students'] thinking by asking them a question that maybe would get them on track." Professor C asked questions as written feedback because "it's something I think a student ought to think about." Therefore, professors asked questions as written feedback to encourage students' thinking with the intent of students arriving at the answer on their own.

Answering questions

The professors provided written answers to 6% of the questions to ensure their students learn from their errors. Professor C claimed, "Well, I think it's just a form of, of telling them, 'You've got something wrong, and here's the direction you should have gone.'" Professors A and B claimed that they answer their own question when they observed students repeatedly making the same error. Professor B concluded, "Maybe if I think there's something particularly tricky going on, or maybe if I think a particular student is persistently making the same kind of error, then I wanna make sure they understand what I'm trying to say." Professor A emphasized that she may answer questions "to help them think about what they might have done incorrectly." Thus, the professors believe they answered questions to ensure the student gets an answer.

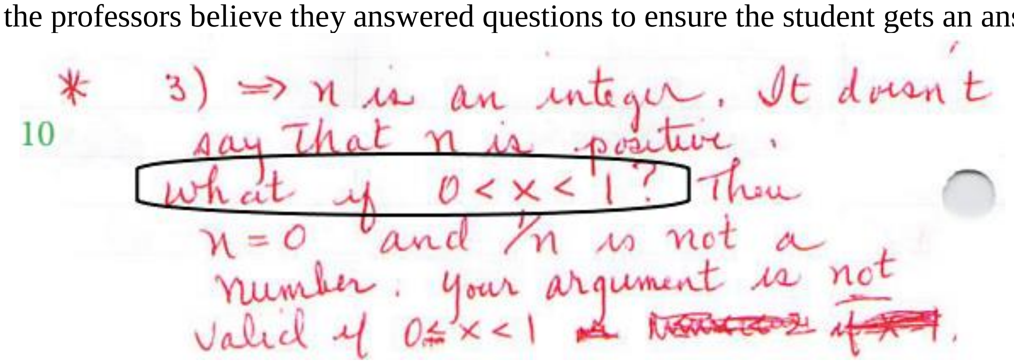

Figure 2 An example of asking and answering a question, attributed to stream of conciousness.

Some of the professors described asking and answering questions in a stream of consciousness. On a homework assignment in Professor D's class, a student used the variable *n* as a positive integer, even though the task needed to be proven for any integer. Professor D questioned the student’s assumption by asking, “What if $0 < x < 1$?” as seen in Figure 2, and explained to the student what would happen if that were true, thereby answering the question for the student. Professor D examined his practice, and said

> So, I guess I could have asked him to think about a case where it's not true. But, see, actually, when I mark a statement like this, I just naturally, as I read this, I, myself, give a counterexample to show that the student's logic is not true. So I guess I just put it down on paper. Might have been just a gut reaction to write that. I don't know if it implies that I wouldn't trust the student to create their own example, to show that what they've written is not true. I guess it does. Because maybe if it was an easy one, I would just say, 'Figure it out.' But here, I thought I should maybe say a little bit more.

Here, Professor D gave multiple reasons for why he chose to answer the questions he posed to students. First, he explained that answering his own question is not always intentional, but a “gut reaction.” Professor D also reasoned that answering the question is also appropriate, if it is probable that the student cannot come to the solution on their own. Finally, Professor D concluded that answering the question allowed the student to see the correct solution.

In general, professors answered the questions they pose to improve students’ proof production skills. Specifically, professors answered their questions to draw attention to an error with the expectation of the student not making the same mistake again, and to emphasize course concepts. Finally, the act of asking and answering the question may simply be a stream of consciousness. Regardless of the reason for answering the question, professors asked questions as written feedback to enhance students’ mathematical understanding and proof writing skills.

Discussion

Written feedback is a common instructional practice in upper-level mathematics courses to help students improve their proof writing (Moore, 2016). In this paper we investigated the practice of leaving written feedback in the form of a question. We found five clusters of questions: *drawing attention to details, seeking explanations, questioning assumptions, structuring the proof,* and finally, *expressing confusion*. Additionally, we presented two explanations for why the professors leave feedback in the form of a question. First, the professors asked questions to prompt the students to think, including training the students to ask questions themselves. Second, the professors claimed the questions mirror their thought process and personal grading style, including the desire to communicate corrections less harshly. Finally, we presented two explanations for why professors occasionally answer the questions they ask as feedback: to ensure students learned from their errors by having access to the answers to the questions, and because the professor asked and answered the question for themselves during the marking process.

The findings of this study are consistent with the research on oral questioning in the classroom as the professors asked written questions for the same reasons they asked questions in classrooms, specifically to probe students’ thoughts and to encourage reorganization of students’ thoughts (Ellis, 1993; Martino & Maher, 1999). Thus, professors asked questions as written feedback for student self-reflection and to promote higher-order thinking.

Questions as written feedback have limitations in their usefulness because the questioning sequence is incomplete. The students were not asked to revise and resubmit their proofs in any of the classes in this study, and as such, the responding and reacting move did not occur. The incompleteness may explain why the professors occasionally chose to answer their own questions; they desired to complete the moves of the questioning game.

References

Acar, F. E., & Kilic, A. (2011). Secondary-School Teachers' Questioning Activities in Learning-Teaching Process. *Education*, *132*, 173-184

Almeida, P. (2010). Questioning Patterns, Questioning Profiles and Teaching Strategies in Secondary Education. *The International Journal of Learning: Annual Review, 17,* 587-600. doi:10.18848/1447-9494/cgp/v17i01/46852

Amrhein, H. R., & Nassaji, H. (2010). Written corrective feedback: What do students and teachers prefer and why? *Canadian Journal of Applied Linguistics, 13,* 95-127.

Brown, E., & Glover, C. (2006). Evaluating written feedback. In C. Bryan & K. Clegg (Eds.), Innovative Assessment in Higher Education, (pp. 81-91). New York, NY: Routledge.

Burns, M. (1985). The Role of Questioning. *The Arithmetic Teacher,* 32(6) , pp. 14-16

Byrne, M., Hanusch, S., Moore, R. C. & Fukawa-Connelly, T. (2018). Student Interpretations of Written Comments on Graded Proofs. *International Journal of Research in Undergraduate Mathematics Education, 4,* 228-253.

Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches.* (3rd edition). Thousand Oaks, CA: Sage.

Ellis, K. (1993, Feb). *Teacher questioning behavior and student learning: What research says to teachers*. Paper presented at the annual meeting of the Western States Communication Association, Albuquerque, NM.

Evans, C. (2013). Making sense of assessment feedback in higher education. *Review of Educational Research, 83,* 70-120. doi:10.3102/0034654312474350

Glover, C. & Brown, E. (2006). Written feedback for students: too much, too detailed or too incomprehensible to be effective? *Bioscience Education, 7,* 1-16. doi:10.3108/beej.2006.07000004

Lew, K. & Mejía-Ramos, J. P. (in press) Linguistic conventions of mathematical proof writing at the undergraduate level: Mathematicians' and students' perspectives. *Journal of Research in Mathematics Education*. Retrieved from the author.

Martino, A. M., & Maher, C. A. (1999, 03). Teacher Questioning to Promote Justification and Generalization in Mathematics: What Research Practice Has Taught Us. *The Journal of Mathematical Behavior*, *18*, 53-78. doi:10.1016/s0732-3123(99)00017-6

Miller, D. A., Infante, N., & Weber, K. (2018). Mathematicians' grading of proofs. *Journal of Mathematical Behavior, 49,* 24-34.

Moore, R. C. (2016). Mathematics professors' evaluation of students' proofs: A complex teaching practice. *International Journal of Research in Undergraduate Mathematics Education*, 2(2), 246-278. doi:10.1007/s40753-016- 0029-y

Mulliner, E. & Tucker, M. (2017). Feedback on feedback practice: perceptions of students and academics. *Assessment & Evaluation in Higher Education, 42,* 266-288. doi:10.1080/02602938.2015.1103365

Price, M., Handley, K, Millar, J. & O'Donovan, B. (2010). Feedback: all that effort, but what is the effect? *Assessment & Evaluation in Higher Education, 35,* 277-289.

Rowe, M. B. (1986). Wait time: Slowing down may be a way of speeding up! *Journal of Teacher Education*, *37*, 43-50.

Speer, N. M., Smith, J. P., III, & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior, 29,* 99-114. doi:10.1016/j.jmathb.2010.02.001

Vardi, I. (2009). The relationship between feedback and change in tertiary student writing in the disciplines. *International Journal of Teaching and Learning in Higher Education, 20,* 350-361.

Walker, M. (2009). An investigation into written comments on assignments: do students find them usable? *Assessment & Evaluation in Higher Education, 34,* 67-78.