Determining Significant Factors for Relating Beliefs to Lecture

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When trying to examine instructors' instructional practices, specifically lecturing, qualitative studies have indicated the necessity to consider their beliefs. However, there is a dearth of quantitative belief measures specific to instructors of undergraduate mathematics courses. No one specific instrument captures the relationship between beliefs and lecturing. This paper, therefore, attempts to establish a foundation of significant factors for researchers to consider when developing belief measures to predict lecturing. We use pre-existing data from Calculus and Abstract Algebra courses to conduct factor analyses and develop composite variables. We then use multiple regression to examine composites with significant effects on time spent lecturing. Results suggest that beliefs related to a focus on skills and content, knowledge facilitation authority, expectations of student success, and the importance of particular concepts are of particular importance.

Keywords: lecture, beliefs, factor analysis, regression

Within mathematics education research, there has been extensive work focusing on improving mathematics instruction. Much of this research has shown that the type of instructional strategies instructors employ depends on their teaching philosophy, how they feel students should learn the material, along with other factors such as attitudes and content knowledge (Mesa, Celis, & Lande, 2014; Remillard, 2005; Weber, 2004; White & Mesa, 2014; Wilkins, 2008). Research has shown there is an interaction between content knowledge, attitudes, beliefs, and instructional practices (Remillard, 2005; Wilkins, 2008). For example, Wilkins (2008) found that content knowledge had a negative effect on both beliefs and instructional practice concerning inquiry-based instruction, indicating that teachers with more content knowledge had lower beliefs and were less likely to use inquiry-based practices; attitudes had a positive effect on beliefs and instructional practices; and beliefs had a positive effect on instructional practices. Remillard (2005) also found that the type of curriculum instructors implement in the classroom was centered on their teaching beliefs and attitudes (Remillard, 2005). Similar results have been found in post-secondary settings in undergraduate mathematics classrooms (Johnson, Keller, & Fukawa-Connelly, 2017; Mesa et al., 2014; Weber, 2004; White & Mesa, 2014).

By understanding beliefs, researchers are able to gain insight on how to modify instruction. Johnson et al. (2017) took this charge and examined instructors' beliefs and the "nature of instruction" to help explain "why there has been little change" (p. 259) concerning instructional practices. They found that some instructors identified as lecturers but used more student-centered instructional practices and instructors who identified as non-lecturers reported lecturing sometimes during class (Johnson et al., 2017). This suggests that an instructor's instructional practices are a complex system made up of both internal and external factors. These factors may very well be in conflict with one another, causing the instructor to sacrifice one belief for another. This calls for the need of models that can help describe instructors' beliefs and offer more insight into conflicting beliefs. These models can also better explain why instructors may choose certain instructional practices over others. However, this can be a taxing job since beliefs are hard to capture in a way that is predictive. As a result, more research is needed to investigate instructors' beliefs to gain better insight for improving instruction. Therefore, the purpose of this paper is to use existing data to establish a foundation of important factors for others considering developing belief measures. Specifically we ask: what belief factors can be used to predict undergraduate mathematics instruction?

Literature Review

Background of Beliefs and Teaching Instruments

We conducted an extensive literature review, searching for literature that focused exclusively on quantitative analysis of beliefs and practices in the STEM or higher education field. What we found was a dearth of instruments used to capture beliefs regarding teaching. These instruments range in disciplines, focusing on general teaching beliefs to more content specific such as Science and Statistics. However, none of the instruments we examined were specific to the mathematics context. The majority of instruments were general, focusing on teaching style preference (Heimlich, 1990), approaches to teaching (Trigwell & Prosser, 2004), or teaching self-efficacy (DeChenne, Enochs, & Needham, 2012; Tschannen-Moran & Hoy, 2001; Thadani, Breland, & Dewar, 2010). Although some of these instruments were newly developed by the researcher (e.g., Heimlich, 1990; Sampson & Grooms, 2013; Trigwell & Prosser, 2004; Zieffler et al., 2012), most often these instruments were developed by adopting previous instruments (e.g., DeChenne et al., 2012; Justice, Zieffler, & Garfield, 2017; Thadani et al., 2010) or expanding them from the K-12 setting to higher education (e.g., Sunal et al., 2001).

The theme from the results of these studies showed that beliefs are directly linked to instruction, and also are predictors of instructional changes (e.g., Sampson & Grooms, 2013; Trigwell & Prosser, 2004; Thadani et al., 2015). Thadani et al. (2015) used four instruments to measure instructors' beliefs: Implicit theories about teaching, Teaching self-efficacy, Implicit theories of intelligence, and Beliefs about students' learning needs. They found that an instructor's belief that teaching skills cannot change subsequently hinders their willingness to improve (Thadani et al., 2015). Sampson and Grooms (2013), as well as Pelch and McConnell (2016), used the Beliefs about Reformed Science Teaching and Learning instrument to investigate instructor's beliefs about science teaching and learning in relation to reformed-based teaching strategies. Results from both studies showed that instructors typically fell on a continuum, ranging from traditional to reform aligned. They also found that by using those beliefs and offering specific training, instructors were able to change beliefs, and that the greatest changes occurred on items related to situational classroom factors (Pelch & McConnell, 2016). Further examining instructors' reform-based beliefs and instructional practices, Borrego, Froyd, Henderson, Cutler, and Prince (2013) used the Research-Based Instructional Strategies survey and found that the instructional practices employed in class aligned with the instructors' beliefs about how students best learn in a limited amount of time. This study identified a "direct link between instructor beliefs and classroom activities specific to engineering courses which rely heavily on problem-solving" (p. 1468). The researchers also claim that this study provides evidence that instructors resistant change due to time constraints.

One concern regarding all the studies we examined was that none of the instruments used were specific to undergraduate mathematics. Although some, such as the STEM GTA-Teaching Self-Efficacy Scale (DeChenne et al., 2012), were specific to STEM, the instruments were not tailored to the field of mathematics specifically. Research has shown that mathematics is a unique content to teach, as there are many beliefs concerning the teaching and learning of it (Johnson et al., 2017; Weber, 2004). For example, Johnson et al. (2017) note that there is a large debate over whether lecture or reformed-based pedagogy is best for the teaching and learning of mathematics. They also note how it is argued that instructors employ instructional practices simply out of habit or because of their beliefs. Due to this debate, there needs to be an instrument designed specifically for mathematics that captures instructors' beliefs and how that might predict instruction.

Building a New Instrument/Model

Prior research has identified numerous belief factors that may influence instructional practices. As was noted above however, very few of the studies we found were specific to undergraduate mathematics instruction. Without such research, those attempting to capture beliefs as they relate to undergraduate mathematics instruction may face confusion over what sets of beliefs to focus on and how to capture them. This concern becomes especially important if researchers are trying to see what kinds of beliefs may predict openness to instructional change as Johnson, et al. (2017) call for. By knowing what belief factors may relate to instructional practices and how to capture them, the mathematics education community can take steps to use those beliefs as leverage points to examine, predict, and even change instruction to meet the calls for educational reform. The aim of our study then is to provide a baseline for which belief factors to focus on in the undergraduate mathematics context and how to capture them quantitatively.

Method

This report draws on pre-existing data from the MAA's 2010-2012 NSF supported study on the *Characteristics of Successful Programs in College Calculus* (CSPCC) and abstract algebra (AA) instructor surveys. Sonnert and Sadler (2015) identified numerous teaching practices students classified as 'ambitious teaching', with many of these paralleling Saxe and Braddy's (2015) definition of active learning. We looked for parallel questions representing instructors' beliefs in such practices in the CSPCC and AA instructor surveys. Further details of the CSPCC study can be found in Bressoud, Mesa, and Rasmussen (2015) while details on the AA study can be found in Fukawa-Connelly, Johnson, and Keller (2016).

Survey Items and Factor Analyses

There were numerous items of interest relating to instructors' instructional beliefs in the CSPCC (16 initial items) and AA surveys (23 initial items). For use in regression analyses, we wanted to maximize our degrees of freedom and create a more parsimonious model and thus used an exploratory factor analysis to create composite independent variables for each survey separately. Numerous models were run with different number of items while eliminating cross-loaded items. We included 13 and 20 items in our final CSPCC and AA factor analyses respectively. The CSPCC data resulted in a four-factor solution (PROMAX rotated) explaining 54.72% of the variance. The AA data resulted in a five-factor solution (PROMAX rotated) explaining 68.22% of the variance. All items had factor loadings above 0.4. Items that loaded onto the same factor were standardized, with items that loaded negatively being reverse coded. Items were then averaged together to create composite variables representing each factor. The factors and included variables are presented below with their factor loadings in parentheses.

CSPCC data. The variables loading onto the first factor asked teachers to estimate what percentage of their students were prepared for the course (.61), and would pass (-.98), fail (.79), or withdraw (.79). As such, we felt the factor represented *Expectations of student success.* The second factor consisted of the questions: 1) *From your perspective, when students make unsuccessful attempts when solving a Calculus I problems, it is: 0 (a natural part of solving the problem) to 5 (an indication of their weakness in mathematics*; .63*)*, 2) rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Calculus students learn best from lectures, provided they are clear and well-organized* (.78), and 3) rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Understanding ideas in calculus typically comes after achieving procedural fluency* (.55). By examining the descriptive statistics for these items (means of 2.65, 3.77, and 3.76 respectively), we felt these reflected a focus on achieving procedural fluency and covering content before conceptual understanding and thus called the composite *Focus on skills and content*.

The third factor consisted of the questions: 1) *From your perspective, students' success in Calculus I PRIMARILY relies on their ability to: 0 (solve specific kinds of problems) to 5 (make connections and form logical arguments*; .75*)*, 2) *My primary role as a Calculus instructor is to: 0 (work problems so students know how to do them) to 5 (help students learn to reason through problems on their own*; .71*)*, and 3) rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *In my teaching of Calculus I, I intend to show students how mathematics is relevant* (.59). We felt these reflected instructors' beliefs about what conceptions they wanted to portray to their students and thus we called the composite *Conceptions of mathematics.*

The fourth factor consisted of: 1) *From your perspective, in solving Calculus I problems, graphing calculators or computers help students to: 0 (understand underlying mathematical ideas) to 5 (find answers to problems*, -.46*)*, 2) rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *If I had a choice, I would continue to teach calculus* (.68), and 3) rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Familiarity with the research literature on how students think about ideas in calculus would be useful for teaching* (.76). This factor seemed to reflect instructors' interest in teaching and perceptions of resources to aid in their instruction and as such, we call the composite *Teaching and Learning Focus*.

AA data. The variables loading onto the first and second factors related to topics teachers felt they should: 0 (*would not cover*), 1 (*try to teach*), or 2 (*always teach*). The first factor consisted of rings (.84), fields (.82), field extensions (.66), ring isomorphisms (.88), ring homomorphisms (.90), and polynomial rings (.86). The second factor consisted of groups and subgroups (.69), group isomorphisms (.83), group homomorphisms (.86), quotient groups (.83), Lagrange's theorem (.69), and the fundamental homomorphism theorem (.81). Regardless of instructors' position on these topics, we felt that the loadings of these items together as factors represented a *focus on fields and rings* and a *focus on groups,* respectively.

The third factor consisted of the following statements instructors rated on a 4-point scale of -2 (*Disagree*) to 2 (*Agree*): 1) *I think lecture is the best way to teach* (.63), 2) *I think lecture is the only way to teach that allows me to cover the necessary content* (.62), 3) *I think students learn better when they struggle with the ideas prior to me explaining the material to them* (-.80), and 4) *I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves* (.74). Based on the positive and negative loadings of these items, we felt that these questions reflected a focus on who instructors believe should control knowledge facilitation and thus was called the composite *Knowledge facilitation authority*.

The fourth factor consisted of the following statements instructors rated on a 4-point scale of -2 (*Disagree*) to 2 (*Agree*): 1) *I think that all students can learn advanced mathematics* (.94) and 2) *I think all students can learn abstract algebra* (.96). We felt these questions reflected instructors' beliefs about students' learning abilities, paralleling the *Expectations of student* *success* factor in the CSPCC data and thus we similarly called the composite *Expectations of student success.* The fifth factor consisted of items asking instructors to rate how influential instructors' experiences as students (.83) and teachers (.83) were on their teaching on a 3-point scale of 1 (*Not at all*) to 3 (*Very*). These seemed to reflect the personal experiences instructors felt impacted their teaching. Thus, we called the composite *Personal influences on teaching*.

Regression Analysis

For the purposes of this study, we were interested in looking for composites with significant effects on time spent lecturing (as one measure of teaching practice). The dependent variable for our CSPCC analyses had instructors rate on a scale from 0 (*Not at all*) to 5 (*Very often*), the statement *During class time, how frequently did you lecture* (mean= 4.20, SD= 1.16). For the AA analyses, teachers answered on a scale from 0 (*Never*) to 4 (*75-100%*), the question *While teaching, what is the approximate amount of time per class that you are lecturing* (mean= 2.64, SD= 1.09). These are categorical dependent variables (with at least five categories), thus we used multiple regression. For each data set, the dependent variable of the amount of time spent lecturing was regressed on the centered composite independent variables specific to that data set.

In terms of diagnostic tests, the regression analyses resulted in VIF values close to 1 (Table 1), indicating that multicollinearity was not an issue. We tested linearity by fitting a Loess line on the plots of standardized predicted values against standardized residuals and by sequentially entering centered power terms sequentially into separate regression models. We checked homoscedasticity by examining the spread of the plots for irregularities. For the CSPCC data, the spread of the data suggests homoscedasticity was a reasonable assumption while the Loess line and statistically significant quadratic model ($F[4, 424] = 2.61$, $p < .05$) suggests linearity may be an issue. The spread of the AA data suggests homoscedasticity may be a problematic assumption while the curvilinear tests suggest linearity was met. Histograms of residuals and P-P plots indicated normality of residuals was satisfied for the AA data but not for the CSPCC data. We checked for outliers by plotting centered Leverage values against instructor ID, which indicated concerns for the CSPCC data. Taken together, these tests suggest that the results of our regression analyses may be inflated for both data sets and other tests may be more appropriate, particularly for the CSPCC data.

Results

For the CSPCC data, *Expectations of student success, Focus on skills and content, Conceptions of mathematics,* and *Teaching and learning focus* together accounted for 2.8% of the variance in the time spent lecturing and the overall multiple regression was statistically significant ($F[4, 427] = 3.08$, $p < .05$). For the AA data, *Focus on fields and rings, Focus on groups, Knowledge facilitation authority, Expectations of student success, and Personal influences on teaching* together accounted for 37.8% of the variance in the time spent lecturing and the overall multiple regression was statistically significant ($F[5, 161] = 19.58$, $p < .05$). As presented in Table 1, there were statistically significant effects of *Focus on skills and content* on CSPCC instructors' time spent lecturing ($\beta_{focus} = .145$, $t = 2.96$, $p < .05$) as well as statistically significant effects of *Focus on groups, Knowledge facilitation authority,* and *Expectations of student success* on AA instructors' time spent lecturing ($\beta_{groups} = .17$, $t = 2.74$, $p < .05$; $\beta_{authority} = .49$, $t = 7.32$, $p < .05$; $\beta_{expectations} = -.15$, $t = -2.26$, $p < .05$). Thus, the more focused CSPCC instructors were on covering content and imparting basic skills first, the more likely they were to spend time lecturing. For the AA data, the higher expectations AA instructors had for their students, the less likely they were to spend time lecturing. By contrast, the more AA focused on the topic of groups or believed in their role as the driving source for knowledge creation, the more likely they were to lecture.

Table 1. Predictors of Time Spent Lecturing

Variable	b	SE	beta	*t*	Significance level	VIF
CSPCC data (N=432)						
Constant	4.192	0.056		75.119	0.000	
Expectations of student success	-0.117	0.075	-0.075	-1.557	0.120	1.027
Focus on skills and content	0.247	0.084	0.145	2.957	0.003	1.058
Conceptions of mathematics	-0.033	0.087	-0.019	-0.385	0.700	1.050
Teaching and learning focus	-0.070	0.087	-0.039	-0.809	0.419	1.047
AA data (N=167)						
Constant	2.569	0.066		39.064	0.000	
Focus on fields and rings	0.155	0.081	0.120	1.900	0.059	1.025
Focus on groups	0.226	0.082	0.174	2.741	0.007	1.039
Knowledge facilitation authority	0.710	0.097	0.489	7.317	0.000	1.154
Expectations of student success	-0.166	0.074	-0.149	-2.258	0.025	1.120
Personal influences on teaching	0.110	0.086	0.081	1.285	0.201	1.020

Conclusions

The factors that resulted from our EFA may be useful subscales for future work attempting to create surveys of instructors' beliefs . To maintain brevity, we suggest retaining two to three questions per factor. The criterion for choosing items should be based on how strongly the item loads onto a given factor. Specifically, items with loadings of the highest absolute value should be considered representative of the factor they load onto. Taking the AA data for example, if we are to have a subscale on *Knowledge facilitation authority* and want to retain two items, we would retain the questions asking instructors to rate their agreement with the statements: 1) *I think students learn better when they struggle with the ideas prior to me explaining the material to them* and 2) *I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves*, as these two had the highest loadings (in absolute value) of all items loading onto that factor (.80 and .74 respectively).

Our regression analyses suggest that the beliefs of particular importance are those related to a focus on skills and content (before conceptual understanding), knowledge facilitation authority, expectations of student success, and the importance of particular concepts. Focusing on these factors can help researchers create more succinct belief assessments. We acknowledge that these factors are only significant in relation to how much instructors lecture. Other factors may be influential in determining other instructional practices and that is an area for future research. Another peculiar finding was the difference in explained variance of instructional practices between the CSPCC and AA data (with more variance explained for the AA data). This may be a result of including items related to topic priority in the AA data but could also result from belief factors having different effects based on context (as can be seen in the difference in beta values for expectations of student success between both data sets). This notion of beliefs varying by context is paralleled in Leatham's (2006) conception of beliefs. Future research should look into how certain belief factors affect instruction differently in different contexts and formulating subconstructs of content specific groupings of concepts (as done with the AA data) which instructors rate on instructional priority.

Our literature review highlighted a dearth of quantitative belief measures specific to undergraduate mathematics instruction. With the results on hand, we have provided some baseline constructs to measure undergraduate mathematics instructors' beliefs in relation to time spent lecturing and other instruction practices.

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