A Mathematician's Instructional Change Endeavors: Pursuing Students' Mathematical Thinking

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To reform instruction by moving towards student-centered approaches, research has shown that faculty benefit from support and collaboration (Henderson, Beach, & Finkelstein, 2011; Speer & Wagner, 2009). In this study, we examined the ways in which a mathematician's instruction unfolded during his participation in a faculty collaboration geared towards reforming instruction and aligning it with inquiry oriented instruction (IOI) (Kuster, Johnson, Andrews-Larson, & Keene, 2017). Results indicate the participant's mathematics background and research interests influenced how he used student thinking in his instruction. More specifically, when mathematics content specifically aligned with the participant's research interest he often guided students to view differential equations as he did; whereas, when the content was not aligned with his research interest, he was more open to the using his students' thinking to drive the class forward. Implications and future research directions are discussed.

Keywords: Instructional Change, Faculty Collaboration, Student Mathematical Thinking

Over the last decade there have been numerous calls for reform in undergraduate mathematics education (e.g., President's Council of Advisors on Science and Technology [PCAST], 2012). These calls for reform draw on research that has shown the benefits of student-centered instruction (e.g., Freeman et al., 2014). To address these calls, change is needed in the instruction of undergraduate mathematics. For example, *A Common Vision* gave a general call that instruction should move away from traditional lecture as the sole instructional method in undergraduate mathematics (Mathematics Association of America [MAA], 2015).

Given these calls for instructional reform, faculty want to make changes to their instruction. However, research has shown that even when working with research-supported curricular materials, mathematics faculty are often unprepared to undertake the challenge of changing their instruction (Henderson et al., 2011; Wagner, Speer, & Rosa, 2007). Current endeavors are providing mathematics faculty with support needed to change their instruction.

There are also calls for departments and faculty members to collaborate specifically on the pedagogy (MAA, 2011). One research-based method of support is faculty collaborations geared towards collectively improving instruction (e.g., Nadelson, Shadle, & Hettinger, 2013). In particular, researchers are studying how mathematics faculty come to use research-based instructional strategies in their classrooms in the context of faculty collaboration. This study explored the experiences of a mathematician who participated in one such faculty collaboration that addresses the numerous calls for reform in undergraduate mathematics education and instruction. The study addressed the following overarching research question: *1) In what ways does one mathematician's experiences in an online faculty collaboration on inquiry oriented differential equations relate to his instructional practice?* And the following sub research questions: *a) How does his instructional practice unfold over his first implementation of inquiry oriented differential equations and in what ways does it align with inquiry oriented instruction? b) How does his participation unfold in the online faculty collaboration?*

Theoretical Backing and Literature Review

Our study and the instructional strategies we sought to disseminate to the mathematics community are rooted in Freudenthal's (1991) theory that mathematics is a human activity. This is manifested in the instructional design theory of Realistic Mathematics Education (Gravemeijer, 1999) on which inquiry oriented mathematics is based. In this section, we briefly describe this instruction and relevant research on instructional change.

Inquiry Oriented Mathematics

The faculty collaboration focused on inquiry oriented mathematics and instruction. Rasmussen and Kwon (2007) defined inquiry oriented (IO) environments as teaching where students are inquiring into the mathematics, while the teachers are inquiring into the students' mathematical thinking. In this study, we specifically focused on inquiry oriented differential equations (IODE) which has been shown effective for student understanding of differential equations (Kwon, Rasmussen, & Allen, 2005).

Inquiry oriented instruction. In inquiry oriented mathematics, it is clear that the role the teacher plays is important for advancing the mathematical agenda. Kuster et al. (2017) recently defined four focal components of inquiry oriented instruction (IOI): generating student ways of reasoning, building on student contributions, developing a shared understanding, and connecting to standard mathematical language and notation. The focal components of instruction are guiding principles of IOI. It is important to note that the four focal components very rarely occur independently; oftentimes, these components overlap and occur in the complexities of an IO classroom. Further, there are local practices of IOI. The local practices of IOI (see Table 1) are an elaboration on the four focal components of IOI. While the focal components are guiding principles of IOI, the local practices are specific actions that instructors do in an IO classroom.

Table 1. Inquiry oriented instructional local practices (Kuster et al., 2017).

Local Practice	Description
1	Teachers facilitate student engagement in meaningful tasks and mathematical activity related to an important mathematical point.
2	Teachers elicit student reasoning and contributions.
3	Teachers actively inquire into student thinking.
4	Teachers are responsive to student contributions, using student contributions to inform the lesson.
5	Students are engaged in one another's thinking or reasoning.
6	Teachers guide and manage the development of the mathematical agenda.
7	Teachers introduce language and notation when appropriate and support formalizing of student ideas/contributions.

Overview of Faculty Instructional Change

Here we first describe barriers to instructional change and then what the research community knows about facilitating and sustaining instructional change.

Barriers to instructional change. One barrier to instructional change is faculty's knowledge for teaching with student-centered instructional strategies. Research has shown that some faculty lack the necessary skills to enact student-center instruction (Hayward, Kogan, & Laursen, 2015), sometimes because they lack specialized content knowledge relating to instruction and being prepared to respond to student questions productively (Wagner et al., 2007). Further, faculty have stated that student resistance, lack of student buy-in, and student attitudes of school are reasons why they do not use student-centered instruction (DeLong & Winter, 1998). The most often cited environmental reason by faculty to not use student-centered instruction is how much more time it takes than teacher-centered instruction (Henderson & Dancy, 2017). Likewise, faculty say they stray away from student-centered instruction because they have a certain amount of material that needs to be covered over the course of one semester (Hayward et al., 2015).

Facilitating and sustaining instructional change. Henderson et al. (2011) outlined four categories of instructional change strategies that are elaborated on in this section: disseminating curricula and pedagogy, developing reflective faculty, enacting policy, and developing a shared vision. Borrego and Henderson (2014) elaborated on these four categories of change by defining eight change strategies that fit within the framework. Our study considered two of these change categories: scholarly teaching and faculty learning communities. Scholarly teaching is when "individual faculty reflect critically on their teaching in an effort to improve" and faculty learning communities are when a group of faculty come together and "support each other in improving teaching" (Borrego & Henderson, 2014, p. 227). These two strategies can work together to improve undergraduate mathematics instruction.

Methods

This study focused on one participant from an IODE online faculty collaboration (OFC). This qualitative instrumental case study (Stake, 1995) was bounded by the participant's participation in the OFC and his classroom teaching. This work comes from the BLINDED project, which supported university mathematics faculty in shifting their practice towards an IO practice. BLINDED offered three supports: the IO materials (in this case IODE), a summer workshop, and the weekly OFC. Here we first highlight pertinent details on the OFC.

Online Faculty Collaborations

The IODE OFC met weekly during the semester they are teaching IODE, virtually via Google Hangouts to conduct lesson studies that were modified Japanese lesson studies (Demir, Czerniak, & Hart, 2013) led by a facilitator. The main goals of the OFC were to: 1) aid teachers in making sense of the instructional IODE materials, 2) thinking through the sequences of tasks, how students might approach the tasks, how to structure instruction around the tasks to support student learning, 3) assist teachers in developing and enhancing their instructional practice, and 4) develop a safe and supportive community.

Participant

The focus of this study is one participant from the IODE OFC, Dr. DM. The OFC consisted of the facilitator (Dr. GG), two graduate research assistants (GRA1 and GRA2), and five faculty teaching the materials for the first time (Drs. DM, AB, PR, CD, ST). The sampling of Dr. DM was purposeful in nature (Yin, 2013) and there were several reasons for that choice. First, he was and is passionate about his participation in BLINDED and to this day continues with IOI in his IODE classroom. Second, he became a facilitator for the project in future semesters following his participant experience. Furthermore, Dr. DM filmed every class of the semester, which was more than was expected of the other BLINDED participants, affording a plethora of possible data sources and a semester-long look at instruction.

Data Collection and Analysis

Data were collected from Dr. DM's classroom instruction, the OFC he participated in, and two interviews during his participation in the project.

Classroom data. Video data from Dr. DM's classroom were collected. Classroom video data were chosen to match the units covered in the OFC lesson studies (i.e., Unit 6 and Unit 9). In addition to those units, Unit 1-2 as an introductory unit and Unit 12 were analyzed. All units lasted a different amount of time. The IOI framework discussed above (Kuster et al., 2017) was designed to capture IOI in action. Consequently, we used the framework as an a priori analytical framework for coding Dr. DM's classroom instructional practice to answer research question 1a. In particular, we used the local practices (LP) of IOI. The IOI framework also contained "evidences," not shown above, of each LP; these evidences served as codes that were collapsed to each LP. LP1 was not coded for unique observable instances in the data. After the first round of coding, we went back again and revisited analysis logs and made adjustments to the coding as necessary. In this step, we looked for emergent themes from the data.

OFC data. Each OFC was screencast using software. All weeks of the OFC were analyzed except week 6 because the data was corrupted and week 8 because Dr. DM was unable to attend that week (in total 9 OFCs were analyzed). Weeks 1 and 2 were introductory weeks. Lesson study 1 took place over weeks 3-5 and lesson study 2 took place over weeks 6-10. Lastly, a debrief OFC occurred during week 11. All videos were transcribed. To analyze Dr. DM's participation in the OFC we coded the transcripts with a priori codes and frameworks: the role of the speaker (production design from Krummheuer, 2007), the role of the listener (reception design from Krummheuer, 2011), and conversation categories (Keene, Fortune, & Hall, under review). These frameworks were adapted to fit the context of this study and are discussed in the results. In a broad sense, we considered Dr. DM's active versus passive participation.

Interview data. The interview data served as a third data source to relate Dr. DM's experiences in the faculty collaboration to his instructional practice. Furthermore, this data offered Dr. DM's personal perspective on being part of a faculty collaboration. Entrance and exit semi-structured interviews were conducted. All interviews were audio recorded and transcribed. Transcripts of both interviews were open coded (Yin, 2013).

Results

Instructional Practice

Central to IOI is the facilitation of mathematics where students are actively inquiring into the mathematics while the teacher is actively inquiring into the students' mathematical thinking (Rasmussen & Kwon, 2007). Dr. DM's instruction focused predominantly on LP2, eliciting student ways of reasoning and contributions (see Table 2). Dr. DM less often actively inquired into why his students were making such contributions (LP3), used those contributions to push the agenda forward (LP4), and had students engage in one another's thinking (LP5; although this happened frequently in Unit 1-2). Note that frequencies were scaled and rounded to represent the same amount of class time as each unit lasted a different number of days.

Table 2. Frequencies of Dr. DM's Local Practices of IOI.

Practice	Unit 1-2	Unit 6	Unit 9	Unit 12
2	58	52	66	26
3	17	24	16	4
4	17	16	15	8
5	42	26	14	2
6	14	16	6	4
7	3	14	8	2

Table 2 is very telling of Dr. DM's instruction. He was very interested in generating student contributions. While some of the questions asked were ones from the IODE tasks themselves, he often would ask his own questions in his own way as a means to address something that he wanted to focus on or have his students think about. While students had opportunities to engage in others' contributions as they were written on the board, they less often had opportunities to engage in others' thinking, as Dr. DM did not follow up with questions to have students elaborate on their thinking as often. Essentially, after students made contributions, Dr. DM would more often move on. We cannot know for sure if Dr. DM was so in tune with the students in his class and the mathematics itself, that he did actually know why his students were thinking along certain lines. However, LP3 and LP4 are about making explicit to the rest of the class such thinking and thus Dr. DM's LP frequencies were reflective of the fact that he did not often make public his inquiring into student thinking.

Comparison of instructional units. Dr. DM's instruction did not necessarily change from the beginning of the semester to the end of the semester. As discussed across the totality of Dr. DM's instruction his most frequent LP was LP2, eliciting student ways of reasoning and contributions. However, when comparing the four units of analyzed instruction there were contrasts between the units. Namely, the way Dr. DM's instruction unfolded was tied to 1) how and when he used student thinking in his class and 2) his mathematical beliefs, rooted in his mathematical research arena.

First, Units 1-2 and 6 were when Dr. DM frequently (more often than any other unit when comparing across scaled time) engaged students in one another's thinking. In particular, these units were the units where his students' thinking was most at the forefront of the class and he oftentimes used that thinking to advance the mathematical agenda. When student thinking was made prevalent to the rest of the class, Dr. DM's instructional reflected that. For example, when introducing phase lines one student made a claim that the solution will never reach 8 (i.e., an equilibrium solution) and the following 8 minutes focused on that one claim. During that time students were responding directly to each other [LP5] or prompted to do so by Dr. DM [LP5]. Dr. DM asked clarifying questions [LP3] such as "and that assessment was based on what?"

Second, when the mathematics of the unit was associated with Dr. DM's mathematical research interests he would focus on getting students to get to "the way [he] view[s] the mathematics" rather than having his students' work or ideas at the center of the development of the mathematical agenda. Unit 9 dealt with the development of the phase plane which was a crucial tool in Dr. DM's research. The instructional portrait of that unit had the highest amount of eliciting student ways of reasoning and contributions [LP2] and in comparison, a very low frequency of LP3-5 (the other practices associated with student thinking). Many of the questions that Dr. DM asked were of his own accord and not generated from the whole class discussion. Because he knew the mathematics so intimately, he was most interested in getting students to see the mathematics the way he does, rather than letting the mathematics emerge from the students.

Dr. DM specifically discussed in his exit interview how he would want students to view mathematics as he does, in particular, the subset of differential equations closely related to his research field: phase planes.

> *Dr. DM [interview]*: And so, um I see DEs, like that's my goal is for students to be able to start to see that. And for that reason, I have to push that kind of phase plane agenda to start to be able to talk about that. ... By viewing myself as the curator of their discussion and just picking apart things and building towards my mathematical agenda allowed me to inject a lot of my personality back into the course and talk about things that I'm really passionate about. … And that agenda is largely because of the way I see DEs used in my research. Uh, I want students to have a taste of that.

Similarly, in class Dr. DM would point out his bias of use of the phase plane.

Dr. DM [class]: This is my home; phase planes are where I live. ... All of my research is based in the phase plane, in phase space. ... That is a sufficiently strong hint that says I will allow my bias to show and I will promise you many questions on the phase plane on the next celebration of knowledge [Dr. DM's tests]. I can't help it. I find it exciting.

Participation in OFC

Recall the goal of the OFC was to support cohorts of mathematicians as they came to learn about IOI and IODE. Table 3 highlights the participation frequencies based on role and conversation. For the purposes of space, we only discuss active and passive participation here rather than all the more specific roles adapted from Krummheuer (2007, 2011). Additionally, we adapted frameworks from our previous work (Keene et al., under review) but here only include four broad conversation categories rather than each individual conversation topic.

Rather than growth throughout the semester, Dr. DM immediately jumped into the active role in the OFC and that active role was consistent throughout the semester. Similar to his classroom instruction there was not a change but rather how his role looked depended on the content of each OFC. For example, if the week focused on doing mathematics, he rarely authored topics because he simply was partaking in the conversation, however, he was very active in those weeks as he has a real passion for mathematics. Additionally, when the OFC focused on sharing of his videos, he authored frequently those weeks and the conversation focused on pedagogy as he sought advice on, for example, how to speed up his class because he was running out of time at the end. Table 3 highlights Dr. DM's most active role related to pedagogical issues.

Table 3. Frequencies of Speaker / Listener Codes by Participation / Conversation Category.

Conversation Category	Speaker		Listener	
	Active	Passive	Active	Passive
Pedagogical Issues	137	16	82	55
Mathematical Issues	70	6	72	40
Student Issues	63	2	20	23
OFC Issues	97	24	91	156

Discussion and Conclusion

In this section, we discuss how Dr. DM's instruction related to his participation in the OFC. In our analysis we observed numerous relationships, but in this report, we specifically focus on how his mathematics background impacted his teaching and his participation in the OFC.

Dr. DM's mathematics background played a role in how his instruction panned out throughout the semester and how he participated in the OFC. In both cases his mathematical content knowledge (rooted in his background and research interests) was placed on top of his interest in enhancing his pedagogical practice. By that we mean, in his teaching, his view of mathematics sometimes was the view of mathematics that he was guiding his students towards. Likewise, in his participation in the OFC, his mathematical understanding was one of the driving factors for his interest in enhancing his pedagogical practice. Namely, he had a deep geometric understanding of differential equations and sought support on how he can get his students to that same level of awe and understanding. Dr. DM desired to reform his instruction but struggled to put aside his prescribed view of mathematics in lieu of his students' mathematics.

This conclusion supports previous work from Speer, Wagner, and colleagues (Speer & Wagner, 2009; Wagner et al., 2007). In their work, they considered the concept of analytic scaffolding necessary for mathematicians to facilitate whole class discussions in inquiry-driven classrooms. They considered analytic scaffolding to be how one supports the mathematical goals of discussion. They remarked, “Gage’s [their participant] analytic scaffolding ... was met with only limited success, despite his strong understanding of the mathematical content, clear vision of the learning goals for the lesson, and commendable ability to elicit contributions from students” (Speer & Wagner, 2009, pp. 558–559). In this quote, numerous parallels can be made between Gage and Dr. DM. Firstly, both had strong understanding of the mathematical content. Second, both had a clear vision of the learning goals. Third, both were very able to elicit contributions from students. Recall that Dr. DM’s most used IOI LP was LP2, eliciting student ways of reasoning and contributions.

However, there are important distinctions that shed light on this topic and provides discussion for faculty collaborations going forward. Most importantly, it brings to the forefront of discussion the subtle notion of a mathematician’s mathematical content knowledge. In their work, Speer and Wagner noted that their participant had a strong understanding of the mathematical content but that did not help in terms of his analytic scaffolding (i.e., meaning facilitation of discussion). Similarly, Dr. DM also had a strong understanding of the mathematical content across all units. However, the difference lies in the fact that in some units he was able to provide analytic scaffolding, namely, he was able to use his students’ ideas in the class (LP3: actively inquiring into student thinking, LP4: being responsive to student contributions, LP5: engaging students’ in one another’s thinking, LP6: guide the mathematical agenda). Yet, he was more likely to do that when the mathematical content wasn’t his specific research interest. Consequently, we concur with Speer and Wagner and posit that one’s mathematics background is not sufficient to successfully use student thinking in one’s class. Additionally, however, the level to which one understands that content makes a difference in their instruction.

In the case of Dr. DM, his focus, for some of the content from the course, was to get his students to his view of the mathematics. This ultimately leads to a tension between his teaching agenda and inquiry. If in inquiry, student thoughts are central to the development of the mathematical agenda (Kuster et al., 2017), then imposing one’s own view of mathematics does not align with an inquiry perspective. The reason this causes a tension is because being passionate about your research inherently is not a bad thing, nor trying to get your students to see the beauty of mathematics. However, in so doing, one privileges their understanding over that of their students. We know from extant literature that mathematicians often struggle to implement novel teaching (if it is new to them) and in particular struggle with how to respond to and deal with student contributions in a productive and successful way (Wagner et al., 2007). However, this was not an issue for Dr. DM as he was in an OFC supporting his instruction. He never noted that he was unsure what his students were going to do. Yet, he seldom actively inquired into his students thinking. This indicates he either knew what his students were thinking or simply did not probe into their thinking; we cannot know which one.

This area of research is ripe for future investigation. The instruction of undergraduate mathematics courses is a hot button item in undergraduate mathematics education research today. More importantly, the research community still needs to know more about how we can support endeavors to reform instruction, how can they be scaled up, and how do we measure success? In this qualitative instrumental case study, while not generalizable, we can conclude that the OFC supported Dr. DM’s desire to reform his instruction. This work has highlighted how those faculty collaborations can be improved moving forward and most importantly highlights that instructional change is possible if the time and effort are put into it.

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