Graphs as Objects: Analysis of the Mathematical Resources Used by Biochemistry Students to Reason About Enzyme Kinetics

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Interpreting graphs and drawing conclusions from data are important skills for students across science, technology, engineering, and mathematics fields. Here we describe a study that seeks to better understand how students reason about graphs in the context of enzyme kinetics, a topic that is underrepresented in the literature. Using semi-structured interviews and a think-aloud protocol, our qualitative study investigated the reasoning of 14 students enrolled in a second-year biochemistry course. During the interviews students were provided a typical enzyme kinetics graph and asked probing questions to make their reasoning more explicit. Findings focus on students' mathematical reasoning, with analysis indicating students tended to focus on surface features when describing related equations and graphs, which limited their understanding of the chemical phenomena being modeled.

Keywords: Graphical Reasoning, Rate, Chemistry

Introduction and Rationale

Enzyme kinetics is an area of study within chemical kinetics, which focuses on modeling the rate of chemical reactions. Looking more broadly at the literature related to students' reasoning about rate-related ideas and the use of calculus to model physical systems, it is apparent that students need more support learning these concepts (Bain & Towns, 2016; Becker, Rupp, Brandriet, 2017; Castillo-Garsow, Johnson, & Moore, 2013; Rassmussen, Marrongelle, & Borba, 2014; White & Mitchelmore, 1996). Biochemistry education research is an interdisciplinary and emerging field and little work has been done that seeks to understand how students reason about biochemistry topics such as enzyme kinetics, indicating the need for more discipline-based education research that can provide insight into how teaching and learning can be optimized (Singer, Nielson, & Schweingruber, 2012). Especially relevant for enzyme kinetics are Michaelis-Menten graphs, which tersely summarize large amounts of data. However, understanding the information a graph communicates (regardless of context) is not trivial (Carpenter & Shah, 1998; Phage, Lemmer, & Hitage, 2017; Planinic, Ivanjeck, Susac, & Millin-Sipus, 2013; Potgieter, Harding, & Engelbrecht, 2007). Nevertheless, even if individuals are not pursuing careers in science, technology, engineering, and mathematics (STEM), in order to have an informed citizenry that can interact with global social issues, individuals should be able to interpret graphs and other forms of data, and have an understanding of how data is collected (along with the associated limitations inherent with data) (Driver et al., 1996; Driver et al., 1994; Glazer, 2011; Mahaffy et al., 2017; Matlin, Mehta, Hopf, & Krief, 2016).

These considerations are encompassed in the Next Generation Science Standards' definition of science practices, which reflect the combination of skill and knowledge used by scientists to approach problems and provide explanations for phenomena, including: asking questions; developing and using models; planning and carrying out investigations; analyzing and interpreting data; using mathematics and computational thinking; constructing explanations; engaging in argument from evidence; obtaining, evaluating, and communicating information (National Research Council, 2012). It is within this context that we investigate student engagement in science practices, such as productively reasoning about models (Michaelis-

Menten model of enzyme kinetics) and drawing conclusions from data (graphs). This work was guided by the following research question: *How do students use mathematical resources to reason about enzyme kinetics?*

Theoretical Perspectives

The design of this study was informed by the resource-based model of cognition, in which knowledge is conceptualized as a dynamic and complex network of interacting cognitive units called resources (Hammer & Elby, 2002; Hammer & Elby, 2003). Within the resources perspective, knowledge is framed as context-dependent, meaning that students' specific resources may not be activated in a particular context, which helps explain fragmented and non-normative reasoning (Hammer, Elby, Scherr, & Redish, 2005). Here we focus primarily on mathematical resources called graphical and symbolic forms, which involve associating (mathematical) ideas to a pattern in a graph or an equation, respectively (Rodriguez, Bain, and Towns, Submitted; Sherin, 2001).

In a forthcoming paper, we provide a more complete overview of graphical and symbolic forms (Rodriguez, Bain, & Towns, Submitted). Tersely stated, graphical forms involve focusing on a region in a graph and assigning ideas; examples include steepness as rate (the relative steepness of regions in a graph provides information about rate), straight means constant (a straight or flat region in a graph indicates a lack of change), and trend from shape directionality (attending to the general tendency of a graph to increase or decrease) (Rodriguez, Bain, & Towns, Submitted; Rodriguez, Bain, Ho, Elmgren, & Towns, Accepted). In the case of symbolic forms, originally developed by Sherin (2001), the pattern under consideration is called the symbol template and the ideas assigned to the symbol template are called the conceptual schema. For example, consider a rate law, which has the following general form: rate = $k[A]^a$. The symbol template for this expression would be $\square = \square\square^{\square}$, where each of the boxes represents a term. The pattern of terms implies mathematical relationships and represents a combination of symbolic forms, such as coefficient (a constant or factor that adjusts the size of an effect), dependence (the magnitude of the value on the left is influenced by changing the values on the right), and scaling exponentially (a term raised to a value scales or tunes the overall magnitude). Generally speaking, graphical and symbolic forms derive their importance from their role in supporting reasoning about processes and phenomena (Becker and Towns, 2012; Kuo, Hull, Gupta, & Elby, 2013; Rodriguez, Bain, and Towns, Submitted; Rodriguez, Satntos-Diaz, Bain, & Towns, Submitted; Rodriguez, Bain, Ho, Elmgren, & Towns, Accepted; Sherin, 2001).

Methods

The participants for this study were sampled from a second-year undergraduate biochemistry course for life science majors in the spring of 2018. Students were given a $20 gift card for their involvement, and all aspects of this project were conducted in accordance with the guidelines of our university's Institutional Review Board. After the participants were tested on enzyme kinetics, we collected our primary source of data, which involved semi-structured interviews using a think-aloud protocol and a Livescribe™ smartpen (Linenberger & Bretz, 2012; Harle & Towns, 2013; Cruz-Ramirez de Arrellano & Towns, 2014). During the interviews the students were given a Michaelis-Menten graph (provided in Figure 1), which they were asked to describe. This prompt was intentionally open-ended in order to provide a general idea of students' reasoning. Students were also asked follow-up questions to make their reasoning more explicit and additional questions were asked to provide insight into resources students used as they reasoned about enzyme kinetics, such as ideas that are more explicitly emphasized in general chemistry (e.g., *What is reaction order? What are rate laws? How is that related to enzyme kinetics?*). Following transcription of the interviews, the data was coded using the graphical and symbolic frameworks, inductive analysis, and a constant comparison methodology (Strauss & Corbin, 1990).

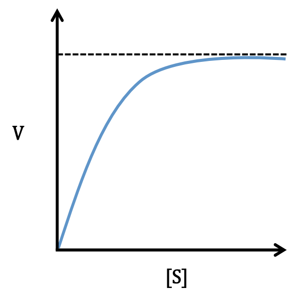


Figure 1. Michaelis-Menten plot provided in the interview prompt.

Preliminary Results

Following analysis we noted student use of mathematical resources was particularly common during (and in some cases isolated to) discussions involving rate laws and reaction order. Generally, students described rate laws in algebraic terms and discussed reaction order in a way that emphasized graphs as objects, affording only a surface-level understanding of the Michaelis-Menten graph provided. However, in some cases, students displayed reasoning that productively integrated mathematical resources and chemistry knowledge, affording a more complete understanding.

Rate Law as Symbol Template

Among the students that discussed rate laws, we observed that the students tended to reason algebraically, which did not productively support their understanding of the Michaelis-Menten model of enzyme kinetics. Looking at the "rate laws" drawn by Tim, Claire, and Alan, we can see there is an attempt to reproduce the rate law by mapping values onto a specific pattern of symbols, which is reminiscent of Sherin's (2001) symbolic forms. In this context, the students were focusing on the symbol template of the rate law and attempting to reproduce some variation of this pattern ($\square = \square\square^{\square}$). This was particularly evident in Tim's discussion where he commented that the rate law for a first-order reaction has two "boxes" (i.e., rate = k[A]), whereas the rate law for zero-order only has one (i.e., rate = k):

> "I think if I remember right, like *k* and then you can do it to like the first order here and then, there was a, yeah, so there was a rate or something was equal to the *k* to the first order … If I remember right … [the rate law] had two boxes for here, but I think zero only had one … because there's two, there's two things that are multiplied here, essentially, you have the enzyme and you have the substrate. And so for the rate you have the enzyme, I think if I remember right for first order you had something multiplied by something else … which would leave for me to think it's a first-order, first-order rate reaction."

Following his discussion of rate laws, Tim then stated that the reaction involving the enzyme and substrate must be first-order, because then the two boxes would be filled by the two reactants. Dorko and Speer (2015) observed a similar "box-filling" tendency when they analyzed calculus students' conceptions of measurement in the context of area and volume calculations, noting that students utilized the *measurement* symbolic form ($\square_{\square}$, magnitude and units), often without considering what values filled the boxes (e.g., 144π as adequate to fill both boxes, even though it represents a single magnitude value).

Graphs as Objects

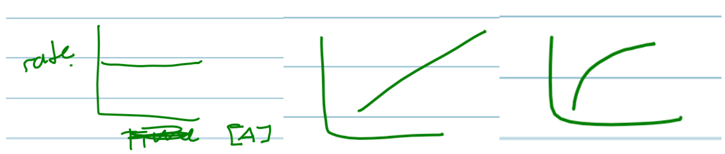
In our dataset the most common conception regarding reaction order involved the association of each order (e.g., zero order, first order, second order) with a particular graph. Eight students in our dataset described reaction order in a way that highlighted the connection between reaction order and graphical shapes, with five of these students explicitly drawing graphs to illustrate this connection. This is analogous to the observed student reasoning about rate laws in the previous section, although in this case the students were focusing on surface-level graphical patterns instead of symbolic patterns. We refer to this type of reasoning as viewing graphs as objects, which is distinct from graphical forms, because in this case the ideas being associated with the graphs are not mathematical in nature.

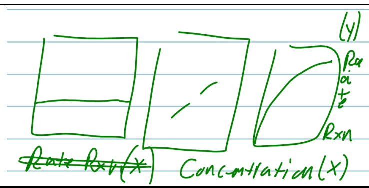
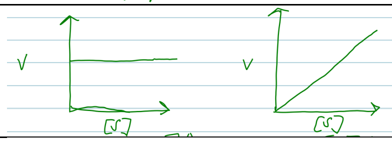
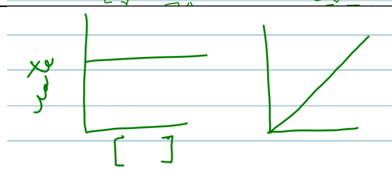
The discussions that accompanied the graphs shown in Table 1 were similar for each of the students, in which they concisely listed the shape associated with each order, focusing on surface features without thinking about the axes (all of the students that drew graphs did not initially draw axes, but some students labeled the axes after prompting by the interviewer, suggesting the salient feature for the students was the shape, and the axes were an afterthought). After discussing the graphical representations of order, the students often attempted to apply shape-centric thinking to reason about the order of the reaction represented in the provided Michaelis-Menten graph, a trend that was observed even for the students that did not draw a graph. For example, in the passage below Amanda discussed the graphs associated with each order, characterizing the Michaelis-Menten graph as a having the second-order “shape”:

> “I believe that's first order, second order, and if it's linear then it's first order or something. If it's just a straight line, it's zero order. … I'm gonna take a straight guess and say it's second order [the Michaelis-Menten graph provided in prompt]. … Because it's curved, and it's ... an exponential … maybe it's a log function, something like that, but I just remember it from the picture that it might be a second order one.”

Although Amanda did not draw graphs to illustrate her understanding, she verbally traced the shapes using reasoning that is consistent with the other students. Amanda’s statement above also provides support for our characterization of students viewing the graphs as objects; in this case the student had an image in mind of the relevant shapes, with which she associated ideas.

Table 1. *Student reasoning about reaction order.*

Student	Written Work
Malcolm	
Karen	

Tim	
Claire	
Carrie	

Conclusions and Questions

The results discussed in this chapter focused on students' ability to connect the Michaelis-Menten model of enzyme kinetics to reaction order and rate laws, which are key tenets of chemical kinetics discussed and assessed in general chemistry (Holme and Murphy, 2012; Holme, Luxford, and Murphy, 2015). As mentioned by Schnoebelen (2018), retention of ideas in general chemistry is higher when concepts are reinforced throughout the undergraduate chemistry curriculum. However, although reaction order and rate laws were discussed in the participants' biochemistry course, they were not the focus of assessment, as is likely the case in other biochemistry courses. Since students study what is assessed, it is not surprising that only a couple of students were able to make the relevant connections, and it should not be assumed that students are making connections between content they are currently studying and content from previous courses (Cooper, 2015). Therefore, we stress the importance of instruction that not only explicitly connects course content (e.g., enzyme kinetics) to relevant concepts previously learned by students (e.g., chemical kinetics), but we also emphasize the role of assessment in student learning, asserting the importance of exams that prompt students to provide evidence they understand these meaningful connections. This work requires further analysis, with the following questions informing our next steps:

(1) What symbolic and graphical forms were productive for reasoning about this context?
(2) How can instruction better support students to make connections between chemistry concepts and mathematical representations?
(3) How do students make connections between the particulate-level mechanism and the graphs/equations used to model enzyme kinetics?

References

Bain, K., & Towns, M. H. (2016). A review of research on the teaching and learning of chemical kinetics. *Chemistry Education Research and Practice*, *17*, 246–262. Becker, N. M., Rupp, C. A., & Brandriet, A. (2017). Research and Practice Engaging students in analyzing and interpreting data to construct mathematical models : an analysis of students ' reasoning in a method of initial rates task. *Chemistry Education Research and Practice*.

Becker, N., & Towns, M. (2012). Students' understanding of mathematical expressions in physical chemistry contexts: An analysis using Sherin's symbolic forms. *Chemistry Education Research and Practice*, *13*, 209–220.

Carpenter, P. A., & Shah, P. (1998). A Model of the Preceptual and Conceptual Processes in Graph Comprehension. *Journal of Experimental Psychology: Applied*, *4*(2), 75-100.

Castillo-Garsow, C., Johnson, H. L., & Moore, K. C. (2013). Chunky and Smooth Images of Change. *For the Learning of Mathematics*, *33*(3), 31–37.

Cooper M. (2015). Why Ask Why? *Journal of Chemical Education, 92*(8), 1273–1279.

Cruz-Ramirez de Arellano, D., & Towns, M. H. (2014). Students' understanding of alkyl halide reactions in undergraduate organic chemistry. *Chemistry Education Research and Practice*, *15*, 501–515.

Dorko, A., & Speer, N. (2015). Calculus Students' Understanding of Area and Volume Units. Investigations in Mathematics Learning, 8(1), 23-46.

Driver, R., Asoko, H., Leach, J., Mortimer, E., & Scott P. (1994). Constructing Scientific Knowledge in the Classroom. *Educational Researcher*, *23*(7), 5–12.

Driver, R., Leach, J., Millar, R., & Scott, P. (1996). Why does understanding the nature of science matter? In *Young People's Images of Science*. Philadelphia: Open University Press.

Glazer, N. (2011). Challenges with graph interpretation: A review of the literature. *Studies in Science Education, 47*(2), 183-210.

Hammer, D., & Elby, A. (2002). On the Form of a Personal Epistemology. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing* (pp. 169–190). Mahwah, NJ: Erlbaum.

Hammer, D., & Elby, A. (2003). Tapping Epistemological Resources for Learning Physics. *Journal of the Learning Sciences*, *12*(1), 53–90. http://doi.org/10.1207/S15327809JLS1201

Hammer, D., Elby, A., Scherr, R. E., & Redish, E. F. (2005). Resources, Framing, and Transfer. In J. P. Mestre (Ed.), *Transfer of Learning from a Modern Multidisciplinary Perspective* (pp. 89–119). Greenwich, CT: IAP.

Harle, M., & Towns, M. H. (2013). Students' Understanding of Primary and Secondary Protein Structure: Drawing Secondary Protein Structure Reveals Student Understanding Better Than Simple Recognition of Structures. *Biochemistry and Molecular Biology Education*, *41*(6), 369–376.

Holme, T., Luxford, C., & Murphy, K. (2015). Updating the General Chemistry Anchoring Concepts Content Map. *Journal of Chemical Education*, *92*, 1115−1116. Holme, T., Murphy, K. (2012). The ACS Exams Institute Undergraduate Anchoring Concepts Content Map I: General Chemistry, *Journal of Chemical Education, 89*(4), 721-723.

Kuo, E., Hull, M. M., Gupta, A. and Elby, A. (2013). How students blend conceptual and formal mathematical reasoning in solving physics problems, *Science Education*, *97*(1), 32–57.

Linenberger, K. J., & Lowery Bretz, S. (2012). A Novel Technology to Investigate Students' Understandings. *Journal of College Science Teaching*, *42*(1), 45–49.

Mahaffy, P. G., Holme, T. A., Martin-Visscher, L., Martin, B. E., Versprille, A., Kirchhoff, M., McKenzie, L., & Towns, M. (2017). Beyond "Inert" Ideas to Teaching General Chemistry from Rich Contexts: Visualizing the Chemistry of Climate Change (VC3). *Journal of Chemical Education, 94*(8), 1027-1035.

Matlin, S. A., Mehta, G., Hopf, H., & Krief, A. (2016). One-World Chemistry and Systems Thinking. *Nature Chemistry***,** *8*(5), 393-398.

National Research Council. (2012). *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas.* The National Academies of Press: Washington D.C.

Phage, I. B., Lemmer, M., & Hitage, M. (2017) Probing Factors Influencing Students' Graph Comprehension Regarding Four Operations in Kinematics Graphs. *African Journal of Research in Mathematics, Science, and Technology Education*, *21*(2), 200-210.

Planinic, M., Ivanjeck, L., Susac, A., & Millin-Sipus, Z. (2013). Comparison of university students' understanding of graphs in different contexts. *Physical Review Special Topics – Physics Education Research, 9*, 020103.

Potgieter, M., Harding, A., & Engelbrecht, J. (2007). Transfer of Algebraic and Graphical Thinking between Mathematics and Chemistry. *Journal of Research in Science Teaching*, *45*(2), 297-218.

Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? *ZDM Mathematics Education*, *46*, 507–515.

Rodriguez, J. G., Bain, K., & Towns, M. H. (2018). Graphical Forms: The Adaption of Sherin's Symbolic Forms for the Analysis of Graphical Reasoning Across Disciplines. Submitted.

Rodriguez, J. G. Bain, K., Ho, F. Elmgren, M. & Towns, M. H. (2018). Covariational reasoning and mathematical narratives: investigating students' understanding of graphs in chemical kinetics. Accepted manuscript.

Rodriguez, J. G., Santos-Diaz, S., Bain, K. & Towns, M. H. (2018). Using Symbolic and Graphical Forms to Analyze Students' Mathematical Reasoning in Chemical Kinetics. Submitted.

Schnoebelen, C. (2018). Evaluation of a Redesigned Chemistry Course Sequence for Undergraduate Life Science Majors. Purdue University: Unpublished doctoral dissertation.

Sherin, B. L. (2001). How Students Understand Physics Equations. *Cognition and Instruction, 19*(4), 479-541.

Singer, S. R., Nielson, N. R., & Schweingruber, H. A. (2012). *Discipline-Based Education Research: Uuderstanding and Improving Learning in Undergradute Science and Engineering*. Washington, DC: National Academies Press. http://doi.org/10.17226/13362

Strauss, A., & Corbin, J. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques*, Newbury Park, CA: SAGE Publications, Ltd.

White, P., & Mitchelmore, M. (1996). Conceptual Knowledge in Introductory Calculus. *Journal for Research in Mathematics Education*, *27*(1), 79–95.