Examining Graduate Student Instructors' Decision Making in Coordinated Courses

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In an effort to improve teaching and learning in undergraduate mathematics courses and help graduate students learn how to teach, many departments across the United States have begun coordinating courses. Although coordination may provide structure and remove some variability in the classroom, there are still many decisions made in the classroom that cannot be coordinated. The purpose of this study was to examine the "uncoordinated" decisions that graduate student instructors made when enacting examples in the classroom. To examine this phenomenon, I studied the cognitive demand of the examples that graduate student instructors chose to enact and the roles that they took on while enacting high cognitive demand examples. As a result, I found that less than 27% of the examples that I observed were enacted at a high level of cognitive demand and that there were three roles (modeling, facilitating, and monitoring) that instructors took on while enacting examples.

Keywords: graduate student instructors, coordinated courses, cognitive demand, examples, decision making

The purpose of this study is to examine graduate student instructors' (GSIs) decision making in coordinated courses. In the department where I conducted my study, precalculus courses are primarily taught by GSIs and are highly coordinated. This coordination involves common lesson guides, student worksheets, WeBWorK homework assignments, and exams. These courses are coordinated primarily by a GSI who serves as the Associate Convener, but there is also a Faculty Convener. Although the high level of course coordination means that GSIs do not have to make many of the decisions regarding course structure and assessment, the lesson guides provided to GSIs allowed them flexibility regarding what examples they chose to do and how they chose to present them. So, for this reason, I chose to examine the examples that GSIs enacted in their classrooms by looking at both the cognitive demand and the roles (modeling, facilitating, or monitoring) that the GSI took on while enacting the example.

Background

The cognitive demand of mathematical tasks is something that has been widely studied in the literature (Boston & Smith, 2009; K. J. Jackson, Shahan, Gibbons, & Cobb, 2012; Kisa & Stein, 2015; Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996). Studies have found that high cognitive demand tasks provide students with more opportunities to learn (Floden, 2002; K. Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Smith & Stein, 1998; Stein, Remillard, & Smith, 2007). Researchers have also found that high cognitive demand tasks are difficult for instructors to enact (Henningsen & Stein, 1997; Rogers & Steele, 2016). But what would it mean to have a high cognitive demand mathematical example? Examples are different from mathematical tasks that are primarily worked on by students. Examples may involve input from students or opportunities for students to work independently or in groups on parts of the example, but usually the teacher plays a leading role in working out or explaining the mathematics. Although studies have shown that students do not learn as much from observing a worked out example as they do from actively engaging in the problem solving process (Richey & Nokes-Malach, 2013), the examples that teachers use still play an important role in the learning process (Chick, 2007; Muir, 2007; Rowland, 2008; Zaslavsky & Zodik, 2007). In particular, Ball and her colleagues (TeachingWorks, 2017) identified "explaining and modeling content, practices, and strategies" as a high-leverage teaching practice.

Methods

The GSIs that I observed (Dan, Emma, Greg, Juno, Kelly, and Selrach) were all experienced graduate students who were teaching precalculus. These GSIs were experienced in two ways. First, they were in at least their third year of graduate studies, had earned their M.S. in Mathematics, and were working towards their Ph.D. Second, they were all teaching their respective course for at least the third time. It is also important to note that many, but not all, of the GSIs had went through a one-year course on *Teaching Mathematics at the Post-Secondary Level*. This 3-credit course was taught by a faculty member in the department who was the Director of First-Year Mathematics. All second-year GSIs were required to take this course in addition to their normal 9-credit course load, but were also given a course release during the fall semester to compensate for the extra time. Alex and Dan were in the first cohort of GSIs who took this course during Year 1. Greg was not required to take this course, but chose to with the first cohort. Emma, Juno, and Kelly were in the second cohort of GSIs who took this course during Year 2. Selrach did not take this course, as it was not offered when he started the program and he did not opt in to take it later. The goal of this course was to support GSIs as they became evidence-based practioners of mathematics education. So, the course aimed to help make GSIs aware of mathematics education research, issues, and terminology so they could apply what they were learning in their own classrooms and become reflective teachers.

For this study, I conducted semi-structured pre-observation interviews, classroom observations, and semi-structured post-observation interviews. I also collected copies of the lesson guides that were provided to the GSIs, the individual lesson plans that the GSIs prepared, and the student worksheets. During the pre-observation interviews, I asked questions about the previous and next class and focused on what examples they planned to use and why. During the classroom observations, I collected video data and took field notes. After each observation, I watched the video and selected one or two examples to discuss with the GSI during the post-observation interview and tagged interesting moments to use for video-stimulated recall.

Each enacted example was first coded using a modified version of Smith and Stein's (1998) framework for the cognitive demand of examples. A full description of this modified framework can be found in Miller (2018), but included four categories for the cognitive demand of examples: *memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics*. Next, I open coded the high cognitive demand examples to examine the roles the GSIs took on while enacting (note that I did not code low cognitive demand examples). Three roles emerged out of this open coding (modeling, facilitating, and monitoring), which I have defined below in Table 1. I then went back and recoded each high cognitive demand example using the final coding scheme for GSI roles.

For this study, I observed each GSI three times throughout the semester. In the first semester, I observed Alex, Greg, and Kelly and asked them to choose three dates (spread out from September-December) that worked best for them. During the second semester, I observed Dan, Emma, Greg, and Selrach and chose specific lessons that I wanted to observe. The lessons that I chose for the second semester were more procedural, because I thought they would provide me with an opportunity to see whether GSIs chose to present examples as *procedures without connections* or *procedures with connections*. Also, I only observed one day of instruction in the first semester, regardless of whether or not the lesson was spread out over two days. However, if a lesson was spread out over two days in the second semester, I observed both days.

Table 1. Definitions of the three types of roles (modeling, facilitating, and monitoring)

Term	Definition
Modeling	An instructor is modeling content, practices, and strategies if they are working through an example independently and expecting students to follow along by taking notes.
Facilitating	An instructor is facilitating a whole class discussion if they work through an example together with input from their students.
Monitoring	An instructor is monitoring if they are requiring students to work through an example independently or in small groups.

Results

Of the 93 examples that I observed, I coded 25 of them as high cognitive demand examples. When enacting high cognitive demand examples, GSIs used a variety of approaches. Although some GSIs took on primarily one role when enacting high cognitive demand examples, others transitioned back and forth between different roles. Figure 1displays the aggregate role profiles for the high cognitive demand examples that I observed each GSI enact. These role profiles were constructed by summing the total time each instructor spent in each role across all of the high cognitive demand examples that I observed and provide a glimpse of which roles each instructor tended to take on. In this paper, I will focus on three role profiles: modeling, modeling and facilitating, and facilitating and monitoring. Although there were several GSIs who enacted examples using these different role profiles, I will focus on specific examples enacted by Emma, Greg, and Kelly in order to illustrate the different ways in which these GSIs chose to enact high cognitive demand examples in their classrooms.

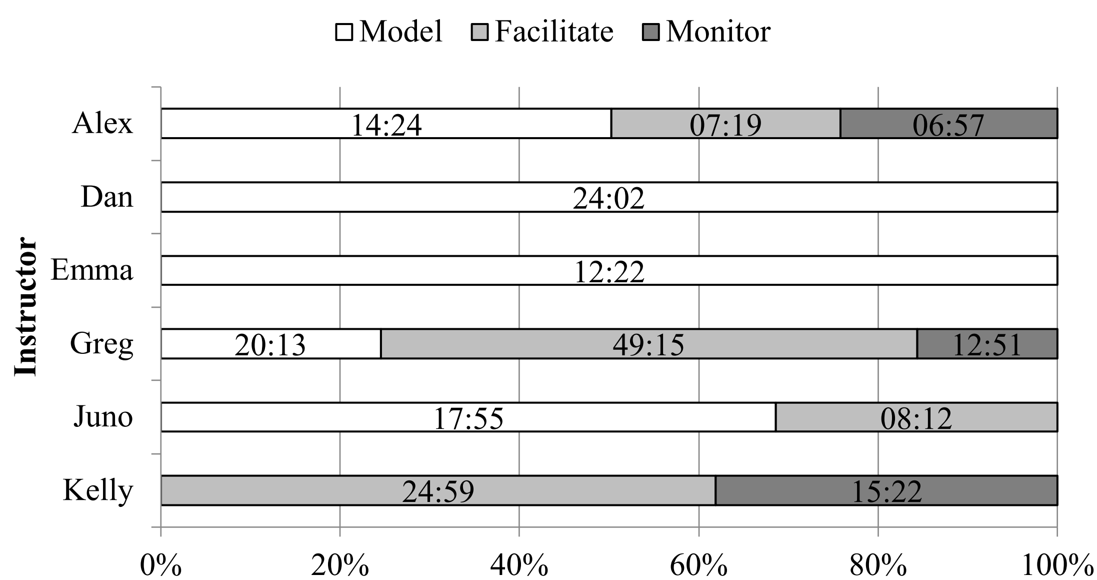

Figure 1. Aggregate role profiles for each GSI

Model: Emma

Many GSIs chose to take on different roles when enacting examples, but some chose to just model examples for their students. Although students do not have an opportunity to struggle with the mathematics in this type of setting, they do have an opportunity to have high cognitive demand processes modeled for them. In order to maintain the cognitive demand while modeling, GSIs focused on making their cognitive processes explicit and attending to student understanding. The example that I observed Emma enact at a high level of cognitive demand was situated at the end of a chapter on function transformations. Emma chose the example because it was a question on the chapter quiz that many of the students had struggled with. In particular, she wanted to reemphasize the connection between order of operations and order of transformations and explain how to check their work using an alternative method. The example gave the graph of a piecewise linear function and asked students to sketch a graph of $3P(t+1)-2$ for $0 \le t \le 9$ on a provided grid.

Since so many of her students had struggled with this problem on the quiz, Emma chose to model it for her students at the beginning of the next class. Emma worked through the example by first identifying the order of transformations. She emphasized that it did not matter if they did horizontal transformations before or after vertical transformations, but that they did need to attend to the order of the vertical transformations. To help her students understand why the vertical stretch had to occur before the vertical shift, she explained how function transformations are related to the order of operations. Next, Emma explained that they could transform the endpoints and corners of the graph and then connect these points with straight lines. Emma also noted that one of the transformed endpoints fell outside the domain $0 \le t \le 9$ and explained how to find the new endpoint. Since so many of her students had struggled with determining the correct order of transformations, Emma also presented an alternative method for graphing the transformed function that did not rely on memorizing information related to order of transformations. Instead, she explained how students could use the equation $3P(t+1)-2$, the original graph, and integer values in $[0,9]$ to graph the transformed function.

I coded this as a *procedures with connections* example because of the following reasons. First, Emma focused students' attention on the use of procedures for the purpose of developing deeper understanding of mathematical concepts and ideas. To help her students remember the order of vertical transformations, she focused on the underlying mathematical concept of order of operations. Also, to help her students find exact output values, she focused on the underlying concept of slope and how to interpret it in a way that is helpful for calculating non-integer values. In her example, Emma presented two different pathways that students could follow to solve the problem (using order of transformations to move points or using an input-output table). In explaining each pathway, Emma focused on the underlying conceptual ideas (order of operations and evaluating function compositions), instead of the narrow algorithms. The example involved graphical, algebraic, and tabular representations and Emma often made connections between each of them. Finally, the number of student questions and the prevalence of student struggle on the problem when it was presented on the quiz are evidence that the example required some degree of cognitive effort for students to follow.

Model and Facilitate: Greg

The high cognitive demand example where Greg switched back and forth between modeling and facilitating was situated in the second day of an extended lesson on finding all solutions to trigonometric equations. After spending the first day exploring the structure of the infinite families of solutions and working through simpler problems that did not involve shifts and stretches, Greg introduced more complicated sinusoidal functions. First, Greg did two examples that only involved vertical transformations. For his final example, Greg chose to find all solutions to $\sin(3\theta - 1) = 1/4$. Greg chose this function for several reasons. First, he wanted his students to learn how to find all solutions when the period is not equal to 2π. Second, he wanted to give an example with both a horizontal shift and a period change because he knew that problems of this type would come up on the online homework as well as the exam. Finally, he did not want to use a standard unit circle angle and instead force students to use arcsine.

Greg started by first modeling content, practices, and strategies for students. To make the equation more clear and appear less complicated, Greg decided to define the variable $X = 3\theta - 1$. Greg chose to do this because he wanted to remove the part of the equation that looked unfamiliar and highlight that first they needed to isolate the input of sine. Next, Greg switched to facilitating a whole class discussion. First, he asked how they could proceed from $\sin(X) = 1/4$ to solve for X. A student suggested that they could use arcsine, so Greg wrote $X = \sin^{-1}(1/4)$ and explained that this gave the first solution. When Greg asked where the second solution came from they were able to come up with $X = \pi - \sin^{-1}(1/4)$ with some assistance from Greg. From here, Greg switched back to modeling. He explained that since they had started with θs, they needed to end with θs and substitute out the Xs. Doing this resulted in the following two equations: $3\theta - 1 = \sin^{-1}(1/4)$ and $3\theta - 1 = \pi - \sin^{-1}(1/4)$. Before solving for θ, Greg paused to explain that this problem "was a little bit more involved than the other [examples] because we generate our initial solutions and then we have to keep working to…find the initial solutions just in terms of θ." From here, Greg worked through the algebra to solve for θ, which resulted in $\theta = 1/3(\sin^{-1}(1/4) + 1)$ and $\theta = 1/3(\pi - \sin^{-1}(1/4) + 1)$.

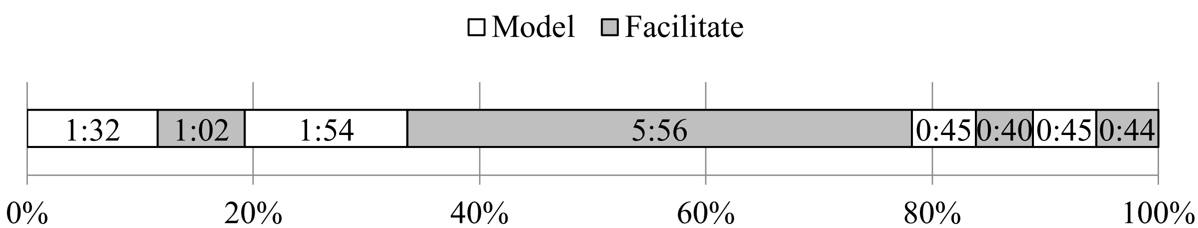

Figure 2. Role profile for Greg's example

At this point, Greg switched back to facilitating by pausing and asking for student questions. Students asked, "Why divide by 3? Where did the $1/3$ come from?" and Greg explained the algebraic step the student was stuck on. Next a student asked, "Will we still involve adding the period times k at the end?" Greg explained that was the next step and reiterated that the work they had done so far was all to get the initial solutions. Greg then moved on to talk about all possible solutions and reminded the class that they should be of the form $(\text{initial}) + (\text{period})k$. To start this conversation, he asked, "What is the period of [$\sin(3\theta - 1)$]?" After working collaboratively, the students were eventually able to identify that the period was $2\pi/3$ and then wrote up the final solutions. Throughout this conversation, Greg switched frequently back and forth between modeling and facilitating. At the end, Greg took the time to summarize the whole process and the general procedure that they had followed.

I coded this example as *procedures with connections* for the following reasons. Although parts of this example strayed into lower cognitive demand tasks, the majority of the problem was focused on the broad general procedure of using the initial solutions and the periodicity of sinusoidal functions to find all solutions. Greg consistently focused students' attention on the underlying structure of solutions to trigonometric equations: $(\text{initial}) + (\text{period})k$. There was a lot of algebra involved in getting the initial solutions and students struggled to find the period, but Greg always brought the focus back to this underlying concept. Although the example was computational, Greg emphasized the connections between the general form of solutions to trig equations and the specific families of solutions that they had found. Also, the number of questions asked by students is one form of evidence to support the claim that this example required some degree of cognitive effort for students to follow.

Facilitate and Monitor: Kelly

The high cognitive demand example that Kelly presented by both facilitating a whole-class discussion and monitoring students as they worked individually or in small groups was situated at the beginning of the lesson introducing exponentials. To start class, Kelly asked her students to work on a problem that asked students to compute the account balances in an account that earned simple interest and an account that earned compound interest. During this time, she asked a group to write the balances in both accounts after one year on the board. After a few minutes, Kelly brought the class back together to see if everyone agreed with what the students had written on the board. She then asked a student to volunteer the balances after two and three years and wrote those on the board. Kelly then asked, “Which one would you chose?” A choral of students said responded with the same answer and Kelly explained why that was correct.

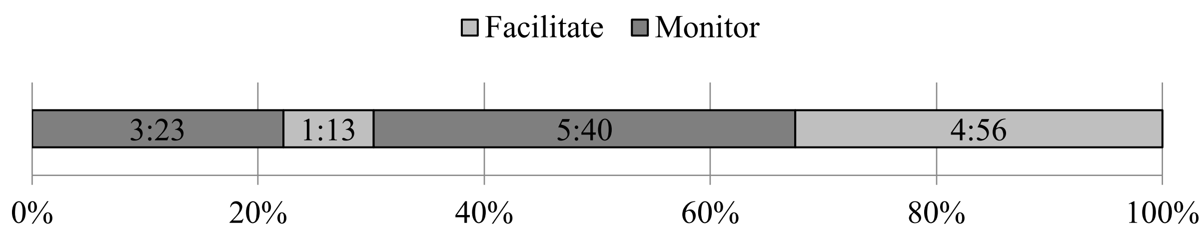

Figure 3. Role profile for Kelly’s example

At this point, Kelly gave her students a similar problem to work on: “Suppose you are investing $\$500$ at an annual rate of 4.5%. Create a table that shows the balance after 0, 1, 2, and 3 years. What is the balance after t years?” As students began working individually and in small groups on this problem, Kelly monitored their progress by walking around the room and interacting with different student groups. After almost six minutes of work time, Kelly brought the whole class back together for a discussion of the general formula. First, Kelly asked students what values they found for the table and verified that everyone had gotten the same answers. Then Kelly asked, “So how are we getting these numbers?” One student explained that they were using the formula $a(1+r)^t$ and Kelly acknowledged that this was correct, but she wanted them to figure out why that formula made sense.

To help start the discussion, Kelly asked, “How did we get from $\$500$ to $\$522.50$?” Another student responded with, “Times 500 by 0.045.” Kelly agreed that this would work, but asked if anyone knew an easier way of doing that. A new student piped up and said, “Times 500 by 1.045.” Kelly responded by explaining how we could factor out a 500 from both terms in $500 * 0.045 + 500$ and get $500(0.045 + 1)$. Next Kelly asked how they had found that $\$546.01$ was the balance after two years. A student responded with, “522.5 times 1.045,” which Kelly agreed with. Kelly asked, “What’s another way of writing 522.5?” After working together, the students were eventually able to refer back to the equation $522.5 = 500(1.045)$. Kelly then explained that to get 546.01, we needed to multiply that again by 1.045 to end up with

$500(1.045)(1.045) = 546.01$. After writing this all on the board, Kelly asked her students if they saw a pattern and if they could guess what the formula for t years would be. A student responded with $500(1.045)^t$. Kelly then encouraged her class to plug in $t = 3$ and verify that the value agreed with what they found in their table. Kelly asked for any final questions, with no response, and then asked, "So what kind of formula is that?" A student responded with exponential and Kelly explained that this is what the new chapter was all about.

I coded this as a *procedures with connections* example for the following reasoning. First, Kelly expected her students to be familiar with exponentials and know how to work with them computationally, but she really focused the example on the underlying concept of multiplicative growth. Students were not provided with any specific pathways to follow and Kelly encouraged them to solve the problem in different ways in order to check their work. Kelly also used tabular and algebraic representations of the problem. Finally, not every student was able to come up with a formula during their small group time, so we know that it required some degree of cognitive effort for students to complete.

Conclusion

In this study, I examined the decisions that GSIs made while teaching in highly coordinated courses. Using my modified framework for the cognitive demand of examples, I analyzed 93 examples that were enacted and found that 25 of them were enacted at a high level of cognitive demand. In these examples, I found that there were three roles that GSIs took on during the enactment: modeling, facilitating, and monitoring. Although some GSIs chose to just model examples for their students (e.g., Dan and Emma), others chose to switch between different roles. Juno also modeled examples for her students, but often asked for student involvement and switched to facilitating. On the other hand, Alex and Greg switched back and forth between all three roles, while Kelly chose to never model and instead just facilitated a whole class discussion or monitored her students as they worked on parts of the example independently or in small groups.

One limitation of this study is that the data I collected focused on the GSI and did not incorporate the student perspective. Therefore, I had to assess the cognitive demand of each example based upon the questions that students asked and the mathematical content of each example. Although I tried to define the four different levels of cognitive demand so that a classroom observer could categorize examples, it was still difficult at times to determine whether or not an example required cognitive efforts for students to follow or understand. Another limitation of this study was that is difficult to determine when an GSI is switching between modeling and facilitating. In particular, facilitating still requires contributions from the teacher, so it can be difficult to determine exactly when an GSI stopped modeling and started facilitating a whole-class discussion. Therefore, the role profiles should be interpreted as having a margin of error any time an GSI switched between modeling and facilitating.

References

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*, 119–156.

Chick, H. L. (2007). Teaching and learning by example. In J. Watson & K. Beswick, *Mathematics: Essential Research, Essential Practice.* (Vol. 1). Mathematics Education Research Group of Australasia.

Floden, R. E. (2002). The measurement of opportunity to learn. In National Research Council, *Methodological Advances in Cross-National Surveys of Educational Achievement* (pp. 231–266).
Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*, 524–549.
Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*, 646–682.
Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School, 18*, 24–29.
Kisa, M. T., & Stein, M. K. (2015). Learning to see teaching in new ways: A foundation for maintaining cognitive demand. *American Educational Research Journal, 52*, 105–136.
Miller, E. (2018). High cognitive demand examples in precalculus: Examining the work and knowledge entailed in enactment (Doctoral dissertation). University of Nebraska-Lincoln, Lincoln, NE.
Muir, T. (2007). Setting a good example: Teachers' choice of examples and their contribution to effective teaching of numeracy. In J. Watson & K. Beswick (Ed.), *Mathematics: Essential Research, Essential Practice.* (Vol. 2). Mathematics Education Research Group of Australasia.
Richey, J. E., & Nokes-Malach, T. J. (2013). How much is too much? Learning and motivation effects of adding instructional explanations to worked examples. *Learning and Instruction, 25*, 104–124.
Rogers, K. C., & Steele, M. D. (2016). Graduate teaching assistants' enactment of reasoning-and-proving tasks in a content course for elementary teachers. *Journal for Research in Mathematics Education, 47*, 372–419.
Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational Studies in Mathematics, 69*, 149–163.
Smith, M. S., & Stein, M. K. (1998). *Selecting and creating mathematical tasks: From research to practice*. Mathematics Teaching in the Middle School, 3, 344–350.
Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455–488.
Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics* (pp. 319–369). Charlotte, NC: Information Age Pub.
TeachingWorks. (2017). *High-leverage practices*. Retrieved June 2, 2017, from http://www.teachingworks.org/work-of-teaching/high-leverage-practices
Zaslavsky, O., & Zodik, I. (2007). Mathematics teachers' choices of examples that potentially support or impede learning. *Research in Mathematics Education, 9*, 143–155.