Investigating Student Understanding of Self-Explanation Training to Improve Proof Comprehension

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Self-explanation is a reading strategy in which readers explain a text to themselves as they encounter new information. Hodds, Alcock, and Inglis (2014) reported proof comprehension gains on students who had been trained to self-explain, when compared to students who had not received this training. We report a multiple case study in which we interviewed undergraduate students in introductory and advanced proof-based courses, to examine their understanding of self-explanation training and their use of this strategy throughout one semester. Preliminary findings indicate that self-explanation made students examine each line of the proof more deliberately, because they knew they would have to hold themselves accountable for figuring out how to explain each line of the proof. However, some students reported almost never using the technique, either because they prioritized the proof techniques demonstrated by their professors, or because they only felt the need to do so with particularly difficult proofs.

Keywords: Self-Explanation, Proof Comprehension, Proof Reading

Introduction

Mathematicians and mathematics educators have stressed that comprehending mathematics text is fundamentally different to comprehending traditional text, and that we need to address the reading of mathematics in undergraduate, proof-based mathematics courses if we want to improve students' understanding of the mathematics texts they are asked to read (Cowen, 1991; Fuentes, 1998; Österholm, 2006). Students have traditionally struggled with proof comprehension in undergraduate math classes. Such difficulties have been documented extensively in the literature and include difficulties attending to the logical structure of mathematical statements (Selden & Selden, 1995), and distinguishing between valid and invalid arguments within proofs (Alcock & Weber, 2005). In response to such difficulties, researchers have proposed various strategies. In this study, we focus on the strategy of self-explanation.

Literature Review

Students spend the majority of their time in undergraduate mathematics classrooms taking notes on lectures in which theorems and their proofs are presented to students by their professors (Fukawa-Connelly, 2012). In turn, professors expect their students to study these proofs outside of class. However, in a survey with 175 mathematics majors and 83 mathematicians, Weber and Mejia-Ramos (2014) found that students had vastly different ideas about the expectations of their proof reading behavior. Most mathematicians expected that students needed to spend more time reading proofs compared to the time expected by students. Additionally, when students did read proofs, they did not report engaging with them in ways that aligned with the reading behaviors of mathematicians that are expected for comprehension (p. 19-20). This implies that there is work to be done on the part of mathematics educators to promote proof reading behaviors that encourage productive reading strategies, such as avoiding attending to surface features in favor of attempting to infer implicit warrants between consecutive lines of proof (Inglis & Alcock, 2012). One option for addressing proof comprehension is to change the format of the proof.

Notable examples of such techniques are Leron's structured proofs (Leron, 1983), Alcock's e-Proofs (Alcock, 2009), and Mason and Pimm's generic proofs (Mason & Pimm, 1984). However, these techniques have been met with little success (Fuller et al., 2015; Roy, 2014; Weber et al., 2012) in terms of gains in student proof comprehension.

Another option for addressing proof comprehension is to change the behavior of the reader. The most prominent example of such a technique in the proof-based literature is called the self-explanation strategy in which readers explain lines of texts to themselves as they encounter new information. It is hypothesized that self-explanation improves comprehension by promoting active integration of new knowledge with existing knowledge, the reevaluation of accuracy and usefulness of mental models, and the coupling of the relationship of actions in a text to overall textual goals (Chi et al., 1989; Chi et al., 1994). The genre of mathematical proof lends itself well to self-explanation due to the importance of logical connectives between lines and the principle-based writing style (Rittle-Johnson & Loehr, 2017). Self-explanation is sometimes accompanied by a training which encourages specific types of explanations and discourages other types of comments. In the case of mathematical proof, preferred explanations are those that promote the integration of prior knowledge with the information in the text, the inferencing of warrants to justify the conclusions drawn in specific lines, and the inferencing of goals and sub-goals of the proof. Self-explanation training discourages non-explanations such as paraphrasing and statements about the reader's affective state ('This is confusing' or 'I get this').

Several studies have promoted the use of self-explanation training (Rittle-Johnson, Loehr, & Durkin, 2017), particularly for participants with low levels of domain knowledge (McNamara & Scott, 1999; McNamara, 2004) because it encourages behaviors that align with the hypothesized benefits of self-explanation. Hodds, Alcock, and Inglis (2014) showed that students who received self-explanation training specific to mathematical proof produced more explanations vs non-explanations and received greater proof comprehension scores when compared to an untrained control group. They also showed, using eye-tracking, that self-explanation training changed students' proof reading behavior. Students who received training spent more time fixating on each line of the proof, and more time focusing on between-line transitions than those who did not. However, these successful students did still produce non-explanations. Thus, although self-explanation training has been promoted in the literature and shown to increase proof comprehension, little is known about the ways in which the training is interpreted by students, how those interpretations impact readers' goals while producing explanations, and what material students retain about the training over time. This study addresses these gaps in the literature by having students describe the ways they used their training to create self-explanations in real time, rank explanations in terms of quality, and describe the features that impact the quality of an explanation. This study provides information about how students consciously use the training to create explanations, and how they interpret information about the types of explanations that theoretically promote understanding and those that should be avoided.

Finally, little is known about the effects of self-explanation training over time. Hodds, Alcock, and Inglis (2014) found that readers retained the benefits of self-explanation training after a few weeks after going through the training only once. Arguably some of the greatest benefits of the training proposed by Hodds et al. (2014) are that it takes up no time on the part of the instructor due to the online format, and that it only takes one 20 minute session of a student's time at home. However, in a meta analysis of self-explanation literature, Rittle-Johnson, Loehr, & Durkin (2017) found a large degree of variability with respect to the longevity of the self-explanation effect. This suggests that more research is needed on the degree of initial scaffolding and the frequency of training required to sustain the self-explanation effect over a long period of time. To date, there are no studies of trained students' self-explanation behavior over the course of an entire semester. Thus, although the students' proof comprehension gains in Hodds et al.'s (2014) study were retained after a couple of weeks from initial training, it is unclear exactly what was their self-explanation behavior. It is possible that students did not consciously self-explain (or that they did it rather poorly compared to right after training), yet retained the benefits of the training in other ways (e.g. through the increased between-line transitions found by Hodds et al.). This study aimed to address this issue by interviewing both novice and advanced students immediately after their self-explanation training at the beginning of a semester (and again at the end of a semester) about their self-explanation behavior and their degree of retention of the training material. This information can help us determine how frequently self-explaining training should be done/discussed throughout introductory and advanced proof-based courses in order to see maximum benefits in student proof comprehension.

Research Questions

The goal of this study was *not* to establish whether self-explanation training is effective and leads to increased proof comprehension. These goals would necessitate an experimental study, and have been addressed by Hodds, Alcock, and Inglis (2014). Instead, the goals of this study were to detail the ways in which self-explanation training is used and understood by participants, and to use those data to generate hypotheses as to the ways in which self-explanation training could be made more effective for different student populations. In particular, the questions motivating this study are: How do novice and advanced students who have received self-explanation training (i) use their training to make decisions when self-explaining a proof (including decisions about the quality of individual self-explanations), and (ii) retain information about their self-explanation training (including how often they report using self-explanation over an entire semester)?

Methods

In order to answer these research questions, we conducted a multiple case study (Bromley, 1986). The descriptive and in-depth nature of these goals necessitate a qualitative interview study, while the desire to address nuances between and within various student populations necessitates a method with students in both introductory and advanced proof-based courses.

Participants

Four students were interviewed at the beginning of the Summer 2018 semester. We were able to bring three back for follow-up interviews. Two students were enrolled in an introductory proof-based course, and two in a real analysis course for which the introductory course is a prerequisite. The real analysis course will be referred to as an advanced course for the sake of clarity. Neither of the researchers were teaching these courses in the Summer 2018 semester.

All four participants were men in their second or third year of study at a four-year institution in the United States. Throughout this report, pseudonyms will be used to discuss each participant. Andrew and Brandon were second-year students enrolled in the introductory course, while Colin and David were third-year students enrolled in the advanced course. Both Colin and David had previously taken the introductory course at the same four-year institution.

The four participants were chosen from a list of students that had expressed interest in the study after it was discussed by the researchers during one class session. In the session, the researchers invited all students to indicate their interest in participating in two paid interviews about mathematical proof reading techniques during the semester.

Procedure

The first interview had four phases. In the first phase, students were asked about their current reading strategies and behaviors when reading proofs. In the second, students completed the online self-explanation training used in Hodds et al. (2014). In the third phase, students self-explained a proof involving concepts used recently in their respective math classes (e.g. students in the introductory course self-explained a proof about rational and irrational numbers). Self-explanations were followed by a series of questions that asked students to describe how they did or did not use their training to produce their self-explanations, how the training did or did not impact the way they read and understood the proof, and the degree to which they thought their explanations were of high quality. In the fourth phase, students were given pre-written self-explanations for Proof B[1] from the Hodds et al. (2014) study. The explanations were written to intentionally focus on specific features of self-explanations that were either promoted or discouraged during the training. For example, one explanation would involve both inferencing of connections between consecutive lines and paraphrasing. Students were asked to comment on the quality of these explanations and the features they believed increased or decreased their quality.

The second interview had three phases. In the first phase, students were asked to describe their current proof reading habits and whether those had changed over the course of the semester. In the second, students described what they remembered about their self-explanation training, how often they had used self-explanation over the course of the semester, and what factors either promoted or inhibited their use of self-explanation. Advanced students were also asked to describe the ways in which the self-explanation training did or did not change their established proof-reading behaviors. Students were reminded that saying ‘I don’t know’ or ‘I don’t remember’ was an acceptable answer. Students were also reminded that their use of the training did not impact the success of the study, so they could be honest in their responses. In the third phase, students self-explained a proof involving concepts used recently in their math classes. Students were asked the same questions about their self-explanations from the first interview.

Interviews were transcribed and we are using thematic analysis to generate claims about their proof reading behaviors over time, and the effectiveness and impact of the self-explanation training. Namely, we describe how students reported using the training to generate explanations, to form their ideas about the desirable and undesirable qualities of self-explanations, and the degree to which those qualities were present in their own explanations.

Preliminary Results

We are in the process of analyzing these data, and briefly discuss two of the themes that have emerged from our analysis.

On Perceived Impact of Self-Explanation Training

[1] Proof B is a proof of the statement: n is even if and only if $3n^2+8$ is even for n in N.

Every student interviewed indicated that a main effect of self-explanation training on their behavior when reading proofs was that it made them examine each line of the proof more deliberately than they might have before, because they knew they would have to hold themselves accountable for figuring out how to explain each line of the proof. Andrew, for example, said the while explaining,

> I think it's being honest with yourself because it forces you to say 'am I actually learning things, am I actually retaining information in class, am I doing what I have to do?' because it kind of holds yourself accountable.

Andrew found that self-explanation made him more likely to question his own understanding of the proof. David echoed this statement by saying,

> I guess it makes me not try to skip over lines too quickly. Like I was like 'okay I have to explain this I better read it carefully'. So it basically makes sure that you're reading every line and if I don't know something you won't be able to explain it yourself.

Here, David emphasized that self-explanation motivated him to thoroughly examine each line to ensure that he would be able to explain each part of the proof well.

On Using Self-explanation Over Time

In the follow up interviews, professor influence and proof difficulty were large determining factors for the use of the self-explanation technique. Brandon and Colin both expressed that while the training was influential in the moment, its influence over their actions when reading proof weakened when they returned to class. Colin, for example, stated, "How the professor teaches is always being pounded into me whereas what you mentioned, I only talked with you once." Colin considered the possibility that self-explanation was taught in person by his professor and stressed throughout the course,

> It would be something that would always be there in your mind because you might think of "how might the professor want me to do this?" [Students] probably [think] "this is how I'm going to be tested, this is what [the professor] would want on a piece of paper."

Colin felt that he would be more likely to use techniques endorsed by his professor, because he would assume these techniques would increase his chances of doing well on exams. Brandon, on the other hand, said he rarely used the technique with proofs in class because "If I'm reading a proof for the first time I don't generally use the technique unless I'm confused or something" which occurred with about 20% of the proofs he read. For Brandon, the technique was a resource that was only necessary when he didn't understand part of a proof, but this did not occur often.

Questions for Audience

1. All students emphasized that high quality explanations should explain the logic behind each line of the proof *and* why the line is necessary. However, not all students produced explanations that included both of these qualities. How should this be interpreted?
2. Many students often conflated statements about reading and writing proofs. How should we handle claims in which a student is discussing the benefits of self-explanation for proof writing rather than proof reading?

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