Beliefs About Learning Attributed to Recognized Instructors of Collegiate Mathematics

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Six collegiate mathematics instructors, who had all previously won teaching awards, were interviewed about their beliefs on learning. Differences between the beliefs of PhD and non-PhD mathematicians were evident, perhaps connected to the student population with which each worked. Furthermore, the four PhD mathematicians all held very different beliefs about learning and modelled their teaching accordingly. Additionally, each of the four had created at least one teaching analogy for himself (climbing instructor/spark, showman/coach, Sherpa, facilitator) that spoke to the role he saw himself in within the classroom.

Keywords: Beliefs, Learning, Teaching

Research on teachers' belief systems suggests that there are strong ties between teachers' beliefs and their instructional practices (e.g., Thompson, 1992). Ernest (1989), in particular, identified teachers' beliefs about mathematics and the teaching and learning thereof as key. Since the late 1980s, the work on teachers' belief systems has grown to encompass work on college instructors' beliefs systems (e.g., Bruce & Gerber, 1995, Warkentin, Bates, & Rea, 1993) and now includes a noticeable subset focused on mathematics instructors' belief systems, such as LaBerge, Zollman, and Sons' (1997) interviews with 26 mathematicians, Weber's (2004) documentation of a mathematics professor's teaching style and beliefs, and Speer's (2008) documentation of a doctoral student's beliefs about the learning of mathematics.

This study focuses on the beliefs about learning attributed to mathematics instructors—regardless of whether they held a PhD in mathematics—who had received institutional teaching awards. Although we recognize that receiving a teaching award may not imply teaching excellence, teaching award winners represent teaching role models sanctioned by mathematics departments: They represent what is currently valued by mathematics departments. Thus, we believe that understanding their belief systems, particularly about learning mathematics, is a worthwhile endeavor. To this end, the research question that this study attempts to address is: What consistencies or inconsistencies exist in the attributed beliefs about mathematics learning of award-winning instructors of collegiate mathematics?

The theoretical perspective we espouse aligns with Speer's (2005) view that all beliefs are attributed to teachers by researchers, for we agree that differentiating between professed and attributed beliefs ignores the role of the researcher. Furthermore, we follow Speer in viewing interviews as insufficient for gaining a complete picture of an instructor's beliefs. Thus, we see this study as the opening act to a larger study that collects classroom data and allows for fine-grained levels of investigation.

Method

Due to the small pool of recent teaching award winners, we opted to conduct semi-structured interviews with each of the participants. These interviews provided us with the opportunity to ask a number of core questions regarding the instructors' beliefs, while also allowing the participants to expound on anything they brought up as relevant to their teaching. Lastly, careful reading of the transcripts by both authors allowed for the creation of a coding scheme that enabled us to differentiate and categorize different beliefs about learning, teaching, and mathematics in general.

Participants

Six mathematicians at a large Midwestern university were asked and agreed to take part in a study on beliefs. Each of these mathematicians had received at least one institutional teaching award. Of four possible teaching awards, three encourage nominations from faculty, staff, and students and are decided on by committees at the departmental or collegiate level. These committees are composed of faculty members and sometimes students. The last of the four teaching awards requires nominations from chairs and directors, encourages support letters from fellow colleagues, and is decided upon by a collegiate-level committee. Of the six mathematicians, five received at least one of the three former awards, and one received the latter.

Four of the participants held PhDs in mathematics while the other two did not. One of the non-PhD mathematicians was female, and the other five participants were male. Although unintended, this gender ratio approximately mirrored the gender ratio of their mathematics department at the time. Henceforth, we refer to the two non-PhD mathematicians as Aleph and Beth, and the others as Gimel, Dalet, Waw, and Zayn. (All names are pseudonyms.)

Data Collection

Each mathematician was asked to participate in about an hour-long semi-structured interview on their beliefs about learning mathematics. The interviews were conducted by the first author in the first half of 2018, audio-recorded, and transcribed. All participants were asked several core questions about their teaching, what learning (the action) and having learned (the state) meant to them, the roles of student and teacher in the learning process, differences—should they see any—between learning mathematics and other subjects, the goal of learning, and their own learning.

1. *Learning*:
   a. *Abstract*: Abstract discussion of learning (e.g., learning means to …)
   b. *Self*: Discussion of participant's learning (e.g., I learned by …)
   c. *Student*: Discussion of students' thoughts on learning (e.g., students think that learning is …)
   d. *Learner*: Discussion of properties of a learner (e.g., a good learner has property X …)
2. *Teaching*:
   a. *Abstract*: Abstract discussion of teaching (e.g., teaching means to …)
   b. *Self*: Discussion of participant's teaching (e.g., when I teach, I like to …)
   c. *Student*: Discussion of students' thoughts on teaching (e.g., students believe that teachers are supposed to …)
   d. *Teacher*: Discussion of properties of a teacher (e.g., a bad teacher is someone who …)
3. *Mathematics*:
   a. *Abstract*: Abstract discussion of mathematics (e.g., mathematics is a set of rules …)
   b. *Self*: Discussion of participant's relation to mathematics (e.g., I have always loved mathematics …)
   c. *Student*: Discussion of students' thoughts on mathematics (e.g., students think that mathematics is just a bunch of rules …)
   d. *Mathematician*: Discussion of properties of a mathematician (e.g., a mathematician is someone who …)
4. *Miscellaneous*:
   a. *Context*: Discussion of anything that adds context to the interview, typically factual (e.g., Calculus I used to be taught by X in format Y …)
   b. *Other:* Discussion that does not directly match any of the previous codes but might still be considered relevant (e.g., employers look for people who can solve problems)

Figure 1. The coding scheme used to code the interview transcripts. This scheme illustrates both the top-level codes as well as the subcodes.

Data Analysis

As the interviews were spaced out over several months, we were able to reflect on the interviews before beginning the coding process. We realized that although the focus was on beliefs about learning, the conversations in the interviews also turned toward teaching and the nature of mathematics. Furthermore, participants would tell anecdotes or relate factual statements that fell in neither of these three domains. Thus, we agreed on four top-level codes: *Learning*, *Teaching*, *Mathematics*, and *Miscellaneous*. Furthermore, after doing a trial-run of coding on the first interview, we discerned that there was more nuance to the interviews that was not captured by our four top-level codes, and so we decided to add subcodes to each of them. Figure 1 provides a list and explanation of the codes. The examples in Figure 1 are made up by the authors, as the actual coded segments from the interviews would be too long to include. All transcripts were coded with the qualitative data analysis software MAXQDA.

To stay true to the spirit and flow of the interviews, as well as to make codes coherent, codes typically span multiple lines and include surrounding interactions between the interviewer and interviewee to present the complete context of each coded segment. Furthermore, declarative statements explicitly indicating our participants' beliefs as well as participants' succinct summaries of their own responses were separately highlighted.

Results

We shall discuss three results: (a) Depending on their student population, there was a large difference between the ways in which the instructors spoke about learning and teaching; (b) beliefs about learning mathematics were quite varied among the PhD mathematicians; and (c) teaching beliefs appeared to be tied to the PhD mathematicians' learning beliefs.

Student Population May Matter

In listening to the participants, it became clear that there was a big difference in the way PhD and non-PhD instructors spoke about students, students' learning, and teaching. This was possibly due to the student populations they respectively worked with: The non-PhD instructors typically did not teach proof-based classes and did not exclusively teach mathematics majors—if at all. Furthermore, their courses were often large classes of freshman and sophomores.

In the interviews with the two non-PhD instructors (Aleph and Beth), efficiency and students' motivation were of much larger relevance than in the other four interviews. For instance, Aleph noted that students' "motivation is primarily this piece of paper, primarily getting this grade." Beth echoes these thoughts sharing that a lot of her students lack curiosity and that "If you're gonna come to a university like this, you need to understand the point of it, and I doubt, they don't. They think of it as a stepping stone to a job." Both Aleph and Beth work with students who they perceive as possessing neither motivation nor curiosity for mathematics and who are instead driven by the prospect of a degree and its impact on their career paths.

With the lack of intrinsic motivation being such a concern for Aleph and Beth, it is, perhaps, not very surprising that they embrace efficiency. Aleph clearly stated that efficiency is his "theme", and both speak of the need to save time in lectures. This is achieved by preparing course materials containing a lot of text that students traditionally would have had to copy down. Thus, the perceived lack of student motivation as well as the fixed amount of material that needs to be covered result in a push for efficiency. Aleph summarized this teaching predicament by comparing the teacher-student dynamic to an optimization game in which instructors attempt to maximize students' exposure to content to achieve learning, whereas students seek to minimize their exposure to content to the minimum level required to obtain their desired grade.

Beliefs About Learning Vary Among the PhD Mathematicians

Rather remarkable was the extreme variation of learning beliefs among PhD-holders (Gimel, Dalet, Waw, and Zayn). Gimel stated that learning happens mostly when students are by themselves and solve exercises. Although one might pick up a high-level concept from a group or get a hint from others, "mathematics, it really is a [pause] ultimately a solitary activity." Thus, classes are merely an introduction to the exercises, where the "real learning" happens.

Unlike Gimel, Dalet believed that "Certainly, there is times where mathematics is a solitary activity and, uh [pause] but there is also times when mathematics is a very social activity." This more balanced approach is based on Dalet's belief that one should spend some time figuring things out for oneself, but that there is also much to learn from communication with others—not only by listening, but also by explaining.

Almost antithetical to Gimel's were Waw's views. He declared that "[collaborative] learning is a, in some ways the most effective way of learning." Waw later added: "What's essential is that the student must attempt to formulate their own arguments and in addition they need to, uh, be willing to examine other people's arguments with a critical eye." The examination of other people's arguments and the consequent discussion is an aspect of learning that sets Waw's beliefs apart from Gimel and Dalet's. Thus, these three instructors form a spectrum that ranges from learning is a solitary activity (Gimel) to learning is a collaborative activity (Waw), via a blend of these two (Dalet). Interestingly, these views lined up with the ways in which the three instructors themselves had learned and continued to learn.

Zayn added another layer to the solitary-to-social spectrum by pointing out that learning proof-based mathematics involves students overcoming a hurdle consisting of the details and rigor required in proof-writing. How do they overcome this hurdle? "I think the point is they overcome it, there's as many ways of overcoming it as there are students. And the point is that [pause] if you don't try to, like, force them to do it your way, but you just create an environment where they can do lots of trial-and-error …" He is forthright about having a learning style which is uniquely his own and which he seeks not to impose upon his students. Although Zayn is alone in clearly distinguishing his learning from his students' learning, it should be noted that Dalet, in the middle of the solitary-to-social spectrum, made some remarks in a similar vein: In speaking about overcoming mathematical struggles and getting unstuck, Dalet stated that he does not know how to tell students how to go through that process. "I don't even know if we all do it the same. You know, I assume we don't, you know."

Consequently, each of the four PhD mathematicians had a set of beliefs about learning mathematics that clearly set him apart from the others. These beliefs can be said to vary along two axes: first, from learning mathematics is a solitary activity to learning mathematics is a collaborative activity and, second, from not differentiating between one's own and students' learning to making that distinction.

Connections Between Learning and Teaching Beliefs of the PhD Mathematicians

A particularly interesting theme in the interviews is how closely the PhD mathematicians' beliefs about teaching aligned with their beliefs about learning. All of them had, to different extents, even developed analogies of their roles in the classroom.

Gimel, who saw learning as a solitary activity best achieved through exercises, described himself both as a climbing instructor and a spark. It was his goal to point students towards the exercises he carefully crafted, but it was the students' responsibility to do them and learn from them: "The teacher's job is to lead the student to a convenient rock face that he can climb, and then the student has to climb it." Gimel could be the spark, but the students needed to be the fuel.

Dalet, believing that a balance of solitary and collaborative activity might be ideal, saw himself as a showman and coach. He described his classes as a performance in which he tells jokes and jolts people awake: "I get pretty pumped up, I feel the adrenaline before going to class and I think it comes out, you know, I act pretty excited about what I'm doing." In addition to providing this showman-like extrinsic motivation, he also tries to foster students' intrinsic motivation by taking on the role of coach and providing encouragement. Although his classes are lecture-based, he incorporates his beliefs about collaborative learning by seeking to make his classes very interactive. He encourages a back and forth with his students and does not bring notes to class as he is prepared to change his plans on the spot. Furthermore, he may sometimes hold a problem class giving exercises, circulating the room, and letting students work together.

Waw, as a proponent of collaborative learning, preferred a flipped environment. He viewed himself as a Sherpa, a tour guide of sorts. He made clear that he's "not a tour guide who says, 'OK, look at this, look at that, that, that, that.'" Waw is willing to make recommendations when asked for them. He's not the agent in the tour guide–tourist relationship; his students need to approach him with their interests and questions about the mathematical realm they are touring. In response, he does not provide answers: He provides suggestions.

Lastly, Zayn sees himself as a facilitator: "I am simply there to facilitate with as *little help* as possible, but also giving as much help as needed …" Assuming that everyone learns differently, it is his goal to create an environment that allows him not to teach the students, but to facilitate their learning. This is done in a setting with minimal lecturing and a focus on group work. He also recognizes that this will not be beneficial for all students. Yet, he believes that he can help the greatest number of people with the environment he creates in his courses.

Thus, we see that Gimel, Dalet, and Waw's beliefs about their teaching align neatly with their beliefs about learning. Interestingly, Zayn realized during the interview that his own learning filtered through his belief that all students learn differently and affected his teaching: "I guess I made that whole teaching thing sort of. I guess it *is* made in my image, now that I think about it. I went through all that trouble of saying I don't want them to learn the way I learned, but now that you're making me say it …" Consequently, it appears as though all our PhD participants' beliefs about their own learning translated to how they spoke about their teaching.

Implications

We see at least three implications of these interviews. First, "undergraduate mathematics instructors" as a group is perhaps too broad a set of participants for a beliefs study as the undergraduate population one teaches appears to provide important context for one's beliefs. Second, the interviews demonstrate that as different as the PhD mathematicians are when it comes to their learning beliefs, most of them are very similar in not distinguishing between their own and others' learning. Third, all PhD mathematicians' beliefs about teaching mirrored their own learning, regardless of whether they distinguished between their own and others' learning. Thus, awareness of the differences between one's own and others' learning does not necessarily translate into awareness of the frequent—as we discovered—similarities between one's own learning experiences and one's teaching beliefs.

References

Bruce, C., & Gerber, R. (1995). Towards university lecturers' conceptions of student learning. *Higher Education*, *29*(4), 443–458. https://doi.org/10.1007/BF01383962

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249–254). London, UK: Falmer Press.

LaBerge, V. B., Zollman, A., & Sons, L. R. (1997, March). *Awareness, beliefs, and classroom practices of mathematics faculty at the collegiate level*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Speer, N. M. (2005). Issues of methods and theory in the study of mathematics teachers' professed and attributed beliefs. *Educational Studies in Mathematics*, *58*(3), 361–391. https://doi.org/10.1007/s10649-005-2745-0

Speer, N. M. (2008). Connecting beliefs and practices: A fine-grained analysis of a college mathematics teacher's collections of beliefs and their relationship to his instructional practices. *Cognition and Instruction*, *26*(2), 218–267. https://doi.org/10.1080/07370000801980944

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A sysnthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York, NY: Macmillan.

Warkentin, R. W., Bates, J. A., & Rea, D. (1993, February). *Discontinuities in science teachers' instructional beliefs and practices across grade levels*. Paper presented at the annual meeting of the Eastern Educational Research Association, Clearwater Beach, FL.

Weber, K. (2004). Traditional instruction in advanced mathematics courses: A case study of one professor's lectures and proofs in an introductory real analysis course. *Journal of Mathematical Behavior*, *23*(2), 115–133. https://doi.org/10.1016/j.jmathb.2004.03.001