Peer Tutors Attending to Student Mathematical Thinking

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Attending to and leveraging student thinking is known to be an effective teaching practice, but little research has been done to investigate the ways in which mathematics tutors attend to student thinking. This study will use the construct of decentering and Ader & Carlson's (2018) framework for analyzing teacher-student interactions to describe the ways in which tutors attend to student thinking in the moment. We will also provide examples of how written reflections and stimulated recall interviews can contribute to a tutor's ability to attend to student thinking.

Keywords: decentering, mathematics tutoring, observable behaviors, student thinking

Mathematics tutoring has been linked to improved grades (Byerley, Campbell & Rickard, 2018; Rickard & Mills, 2018) and increases in confidence and positive attitudes towards mathematics (Duranczyk, Goff, & Opitz, 2006). It is very common for universities in the United States to offer peer tutoring for entry-level mathematics courses such as Calculus (Bressoud, Mesa, & Rasmussen, 2015). While there is evidence that tutoring is effective, there have been few studies investigating exactly what happens in a peer tutoring session at the university level. Tutors working one-on-one with a student have the opportunity to make sense of and build upon students' thinking, however, it is unknown if tutors take advantage of this opportunity.

In the context of teaching, carefully listening to student thinking and using it to inform instructional decisions is considered an effective practice (NCTM, 2000). However, studies have shown that teachers are often not naturally able to implement the professional practice of attending to and building upon student thinking (Wallach & Even, 2005). Mathematics professors even have difficulty with making sense of their students' thinking despite their content expertise (Johnson & Larsen, 2012; Speer & Wagner, 2009). Thus, we cannot expect that it will be typical for tutors to make sense of and use student thinking in the moment without training in these skills. It has been found that dialogue patterns between tutors and students tends to focus on the steps of the solution, with the tutor choosing the path and asking the student to contribute to calculations (James & Burks, 2018; Van Lehn, Siler, Murray, Yamauchi, & Baggett, 2003).

In this paper, we address the research question: What is the nature of decentering among college math tutors who have no training in decentering? We present examples from tutoring interactions in which undergraduate mathematics tutors attend to student thinking at varying levels. We also give evidence that written reflections and one-on-one stimulated recall interviews can help tutors begin to consider mathematical thinking from the student's point of view.

Literature Review

Thompson (2000) describes an *unreflective interaction* as one in which the teacher is not attempting to set aside his/her own understanding to discern the mental actions that are driving the student's behaviors. In this instance he would say that the teacher is operating from a first-order model, projecting his/her own cognition onto the student. In contrast, a teacher who is *interacting reflectively* is actively trying to build a second-order model of their student's thinking by asking questions. The term *decentering*, first used by Piaget (1955), is used by Teuscher, Moore, and Carlson (2016) to refer to the teacher's action of trying to understand a student's

perspective *in the moment*. Teachers who are motivated to decenter believe that the student has some viable set of meanings that contribute to his or her actions, even though the student may have an idiosyncratic understanding (Teuscher, et al., 2016). Thus the teacher actively asks questions and strives to make sense of the student's thinking without assuming that the student's thinking aligns with their own. Decentering has been used to analyze teacher decision making in the moment with Ader & Carlson (2018) and Teuscher et al. (2016) providing examples from authentic classroom interactions.

One way teachers have been trained to focus on student thinking is through discussions of video clips of their own teaching with their colleagues in which they craft questions that the teacher could have asked to probe student thinking. Participation in video clubs of this type has been shown to have an impact on teacher practice (Sherin & Van Es, 2009).

Much of the literature related to decentering has focused on classroom instruction (Ader & Carlson, 2018; Teuscher, Moore, & Carlson, 2016) and not one-on-one tutoring interactions. There are many ways in which tutoring differs from teaching. Undergraduate peer tutors are not content experts and have often not received training in pedagogy, but they have a skill set that is different than instructors. They have been successful students themselves, they know how the mathematics applied their subsequent courses, they can communicate informally with students, and they may be better able to sympathize with the concerns of students (McDonald & Mills, 2018). Since the context of teaching and tutoring are so different, it is necessary to examine what decentering could look like in the context of peer tutoring.

While the classroom is certainly an important part of student learning, much of the learning takes place when students are working through the homework, and interactions with tutors could be just as formative for students. Also, while university professors may be resistant to changing their classroom pedagogy, hourly paid undergraduate tutors can be trained to implement any approach that we deem beneficial to students. Thus, the impact of training tutors on student learning can be more immediate than attempting to change departmental attitudes towards progressive teaching methods.

Theoretical Framework

Decentering occurs in the mind of the teacher or tutor, and thus it is impossible to determine precisely whether or not a teacher or tutor is decentering. However, in order for a tutor to decenter and take on the perspective of another person, the tutor must do more than simply project their own cognition about the mathematics onto the student. Ader & Carlson (2018) focus their analysis on *observable behaviors* of the instructor as an indicator of decentering. We will briefly describe their four levels of interaction and associated observable behaviors.

Interactions in the first two levels of Ader and Carlson's (2018) model describe unreflective interactions when the teacher operates from a first-order model. In these interactions, the teacher does not attempt to understand the student's thinking, but assumes that the student's thinking is identical to his or her own. In a Level 1 interaction, the teacher does not pose questions aimed at understanding student thinking. Teachers engaged in a Level 2 interaction ask questions to reveal student thinking, but do not attempt to understand student thinking and rather guide the student to the teacher's own way of thinking. Levels 3 and 4 are reflective interactions in which the tutor asks questions to understand the student's thinking, taking on his or her perspective and building a second-order model to inform his or her instructional decisions. Level 3 interactions are characterized by the teacher "asking questions to reveal student thinking and then following up on student responses to perturb students in a way that extends their current ways of thinking,"

and attempts to move the student towards his/her way of thinking. In a Level 4 interaction, the teacher is focused on using and developing student's idiosyncratic ways of thinking by posing questions or giving explanations that are attentive to students' thinking.

Methods

The data were collected from a drop-in mathematics tutoring center in a Midwestern research institution. Tutors in this study are undergraduates who are trained to spend 5-10 minutes with each student and move around the room. The students that the tutors are working with are enrolled in a wide variety of classes from college algebra through differential equations. The tutoring center employs 40 tutors who work from 6-12 hours per week.

As part of their training, tutors were required to record a tutoring session, and they had a week to transcribe the session and respond to written reflection questions addressing the student participation level, student mathematical thinking, questioning, and the tutor's listening skills. They then scheduled a 10-15 minute interview with the researcher in which they watched the interaction together and the interviewer went through the session line by line asking questions such as "What do you think the student meant by that comment?" and "Why did you choose to ask that question?" and "What was your goal in giving that example?"

The written reflection and interview served two purposes. First, they gave the tutor a chance to elaborate on his or her in-the-moment decision making. This is helpful for training purposes because it allows the supervisor to better understand the tutor's methods. For this study, it has helped us to triangulate the data to build a better case for our classification of tutor moves. Second, it gives the tutor the opportunity to think critically about what they believe the student might have been thinking, and what the tutor could have done differently.

Several interactions were analyzed and heuristic cases which exemplify varying levels of tutor decentering were selected for presentation. During analysis, we examined the LiveScribe recording of the interaction and the transcript to look for observable behaviors of the tutors that indicated their level of decentering. We then read the written reflections and listened to the audiotaped interviews for further evidence of the tutors' attention to student thinking. We should note that in the same manner as Nardi, Jaworski, & Hegedus (2005), we are classifying episodes rather than tutors. Individual tutors can display varying levels of attending to student thinking even in the same 10-minute tutoring session, thus it is not feasible to label tutors according to their tendency to focus on student thinking.

Results

In the observation data that we collected, we have many examples of tutors leading the student through procedures without asking for the student to express his or her thinking. This is consistent with results in physics tutoring (VanLehn, et al., 2003) and undergraduate mathematics tutoring (James & Burks, 2018). Because we want to examine decentering in the context of tutoring, we focus on instances in which students were asked to explain their thinking.

Here we will present three naturalistic tutoring sessions illustrating varying levels of decentering in a drop-in tutoring environment. We will also give an example of how reflecting on a tutoring session can lead a tutor to think more deeply about student thinking and formulate questions that he or she could have asked.

Episode 1: Tutor Decentering In-The-Moment

In this episode, Abby was working with a student to compute the derivative of $f(x) = x^5(3-x)^6$. Abby had already asked the student if he knew how to take the derivative but the student did not suggest using the product rule, so Abby proceeded to walk him through it.

Abby: So the product rule is the first times the derivative of the second plus the second times the derivative of the first, so we take the first *(writes down* x^5*)* and we take the derivative of the second, so what's that derivative?
Student: ummm… six times three minus *x* to the fifth
Abby: Awesome. But soo… three minus *x* to the fifth. That is not just *x* to the fifth, right?
Student: Right.
Abby: So it has a function inside of it
Student: mhm
Abby: So that means you'll have to do that chain rule
Student: Oh okay, yeah
Abby: Okay so what's the derivative of three minus *x*.
Student: One
Abby: Ok, so… Why would you say one?
Student: Because three turns into zero and *x* the one turns into zero, so it's *x* to the zero, which is one.
Abby: Okay, so not quite, whenever you just have the equation $y = x$. What's that derivative?
Student: uhhh… one.
Abby: Right, so…. *y* prime is equal to one, but if you have $-x$, what would that turn into?
Student: Negative one
Abby: Right, so…
Student: Oh okay… It would be negative one.

When the student gave the unexpected answer that the derivative of $3 - x$ is 1, Abby asked the student to explain his thinking further. She then followed up on the student's response to give examples that perturbed the student's thinking in a way that extended his current way of thinking. In her interview, Abby elaborated on her interpretation of the student's thinking.

Interviewer: And then the student is saying that the derivative of $3 - x$ is 1.
Abby: Mmm-hmm.
Interviewer: And then how did you... what were you thinking about that?
Abby: I'm like, there are so many ways that they could have gotten to 1, so I wanted to know how they got to 1.
Interviewer: So you really ask an open ended question there: "Explain why you said that?"
Abby: And they are like "because 3 turns into 0" that's right, and "*x* to the 1 turns into zero" so I assume that they mean *x* to the first, so they are trying to do the [power] rule, and they are like "that goes down, and so it's *x* to the zero and that's just 1." And so that's technically right, but…
Interviewer: They are missing the sign.
Abby: Yeah.

Interviewer: And so you ask a series of questions, or actually kind of give another example to lead them to that, so… Why did you choose to do that instead of just saying "Oh, you missed a negative?"
Abby: Because a lot of people had been not knowing, like they are like $3 - x$, I just don't, like so many people had been missing that negative because I had done this problem like three times before already. And so, I'm like, "well, if y is x, then…" So, it makes a lasting impression.

Abby asked the student to explain his thinking because she was genuinely curious how he "got to 1." When she listened to his thinking, she interpreted his explanation as a correct application of the power rule and deduced that he is just missing the negative sign in the derivative of $-x$. We can see that in this instance, Abby has the natural inclination that leading a student through a series of examples to perturb his thinking will make more of a "lasting impression" on him than just telling him that he missed the negative sign.

We label this interaction as a Level 3 in terms of Abby's decentering actions because she asked a question to reveal the student's thinking and then followed up with two simpler examples designed to perturb the student in a way that extended his current way of thinking.

Episode 2: No Evidence of Tutor Decentering In-The-Moment

Bernard was working with a student to solve a quadratic equation. We can see that the student explained his first step, but Bernard lead the student in a different direction.

Bernard: Okay so the problem is $3x^2 - 2x - 5 = 0$. So what we tried before was…
Student: Multiplying -5 and 3 to get -15 and getting the factors of that to get it.
Bernard: Right, so that's kind of along the right track. So we want to work on the rational roots theorem, which is we take the factors of the last term which in this case is…
Student: 1 and 5
Bernard: 1 and 5. So I'll put that on top here. So 5 and 1, and we'll put that over the factors of the first term which is...
Student: 3 and 1.
Bernard: 3 and 1 right. And so it could also be plus or minus any one of these values so what we could have is +1, +5, +5/3 and +1/3 does that make sense?

Although Bernard prompted the student to outline his strategy, he did not ask the student to elaborate on his thinking, but evaluated the student's response in light of his own way of thinking. He then posed questions that focused on procedures requiring little thinking on the part of the student. It is unclear what Bernard understood about the student's strategy and what he meant by "you are on the right track." In the interview, Bernard was asked to explain what he thought the student was thinking.

Bernard: So, he, I think like in the very beginning he tried multiplying the negative five and the three together and finding the factors of that.
Interviewer: So, what do you think he meant by that?
Bernard: I think, like, so I mean, that's kind of, sort of... meh... At least he was realizing that he needed something from the last term and something from the first term.
Interviewer: Right.

Bernard: I can't remember if there's a way... like another theorem or something where you do that, but I don't remember what...
Interviewer: Yeah. So, you're feeling like, he knew he needed something from that negative five and something from the three, but he was multiplying instead of dividing?
Bernard: Yeah. Yeah.

As experienced mathematics instructors, we realize that the student could be attempting to transform $3x^2 - 2x - 5$ into $3x^2 + ax + bx - 5$, where *a* and *b* are factors of -15 so that he could factor by grouping. Since Bernard did not probe the student's thinking, it is unclear whether or not the student was attempting to use this strategy. It seems that Bernard was not aware of this method and assumed that the student was incorrectly applying the rational roots theorem, although he acknowledged in the interview that the student may have been using a theorem that he didn't remember. This interaction is categorized as Level 1 because when the student gave a response that did not match with Bernard's own method, Bernard did not ask the student questions to understand the mental actions driving the student's behaviors.

Towards the end of his interview, Bernard said, "Yeah. It made me realize that I do a lot of the talking, and it's not as interactive as I'd like it to be. I still ask questions that make sure they understand what's going on, but maybe having them reproduce what I'm writing or having them write down what I'm saying and see if that maybe clicks with them." This gives further evidence that for this problem Bernard was focused on helping the student to adopt his own way of thinking rather than understanding and building upon the student's thinking.

Episode 3: Tutor Develops Awareness of Decentering

The student was attempting to find the intersection of the lines $r_1(t) = <0,1,1> + t<1,1,2>$ and $r_2(s) = <2,0,3> + s<1,4,4>$. The student had constructed six equations: $x = t$, $y = 1 + t$, $z = 1 + 2t$, $x = 2 + s$, $y = 4s$, $z = 3 + 4s$. Emma led the student to set the *x*, *y*, and *z* equations equal to one another. Then, the following interaction occurred:

Student: So does it want it in terms of *t* then or *s* or what?
Emma: So first let's, um, set them equal to each other and find what our *t* and s values are.
[… tutor and student set up the equations and solve them together]
Student: So subtract an *s*, and it's 3*s* = 3? And *s* = 1?
Emma: Exactly.
Student: So, and then I just use that to find *t*, and get *t* = 3?
Emma: Uh huh. So using that first equation
Student: So is that the point then? That intersects?
Emma: Um.
Student: Or is that, or it's in three dimensions, so…
Emma: So, uh, we, you found your *s* and *t* values, so, if we are looking at where they intersect and where they equal each other, we are looking at a point on both $r_1(t)$ and $r_2(s)$, so that means that if we plug in *t* for $r_1(t)$ or *s* for $r_2(s)$, we can pick which one, uh, we should be getting the same answer. So you just pick one of the variables and plug it back in.

This in-the-moment interaction is classified as Level 2. During the tutoring session, Emma did not ask the student to elaborate on his thinking, but continued to ask questions to lead him through the procedure of the problem. She showed interest in the student's thinking to the extent that it revealed the student's misconception, and then she attempted to guide the student to her own way of doing the problem.

> *Emma:* A lot of what he's been doing before is just solving for the variable, and the variable has been the answer, and so maybe here he was thinking, "the variable, is that the answer? But then those are different numbers, but that doesn't really make sense for an intersection... " I think he was just kind of confused on that.
> *[...]*
> *Interviewer:* So talk about what the student might be thinking there.
> *Emma:* So he's found the s and t variables, and I think he's thinking it's kind of like an x and a y, you know, two dimensional, but then he realizes that we're in three dimensions because we have x, y, and z, and so he's trying to figure out how to turn $s = 1$ and $t = 3$ into an answer.

Emma verified that she thought that the student initially thought that the s and t values were the solution, and then the student expressed cognitive dissonance when he realized that he needed a three dimensional answer. Whether she had this view of the student's thinking in the moment is hard to say, but in the written reflection, Emma said, "I could have asked what t and s are used for and why we're trying to find them." This question would be useful for developing a second order model of the student's thinking, and if she had asked it in the moment, the interaction may have been classified as Level 3. Thus, we can see that the process of going through the reflection afforded Emma the opportunity to construct a question that would further reveal the student's mental actions.

Discussion

The main contribution of this paper is the application of decentering to the context of undergraduate peer tutoring. We have given examples of tutoring interactions that display varying levels of tutor decentering in the moment and we have also shown an example of how the process of writing a reflection and re-watching the recording of their interaction can aid tutors in formulating questions to draw out student thinking. Our analysis of decentering was based on both the observable behaviors and the tutor's reflections. Many of the tutoring interactions that were analyzed for this study were Level 1, but we have presented examples of un-trained tutors decentering in the moment to varying degrees.

A limitation to this study is that the tutors self-selected the session that they wanted to transcribe, so the sessions may not be reflective of their typical tutoring. The stimulated recall interviews were not always consistent, and so some of the tutors may have been prompted more than others and the interviewer may have asked the student to comment on a moment that the student may not have spontaneously commented on. Another limitation includes our inability to determine if the questions constructed by the tutors in their reflections will enable them to decenter more frequently. The interactions that we recorded will be used to refine our reflection questions and design tutor training programs that incorporate real tutoring interactions. We can also design studies to investigate whether a focus on decentering in tutor training can improve tutors' ability to understand and leverage student thinking.

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