Undergraduate Mathematics Tutors and Students' Challenges of Knowing-To Act

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Colleges and universities are increasingly providing drop-in tutorial assistance through institutions' learning or resource centers. In this study, we examine one-on-one mathematics tutoring interactions to discover how tutors naturally respond to student requests for assistance with knowing-to act, where a student may be familiar with a procedure, but not know-to use that procedure in the current situation. We contrast three 5-10 minutes episodes; in the first, the tutor appears not to recognize that the student knows-to. In the second, the tutor prevents the student from needing to know-to. In the final episode, a tutor incrementally narrows the vision of her student until the student knows-to.

Keywords: undergraduate mathematics tutoring, types of knowing

Introduction and Review of Literature

While great effort has been and continues to be exerted to study and improve classroom instruction of post-secondary mathematics, comparatively little research has focused on how students study and learn math outside of class. Increasingly, institutions are providing out-of-class assistance for entry level math courses through learning, resource, and tutoring centers (Bressoud, Mesa, & Rasmussen, 2015). These centers have the opportunity to design and implement tutor training, so there is reason to identify tutors' natural tendencies and discover how those tendencies impact student learning.

Studies of human tutoring interactions in disciplines besides mathematics have identified strategies tutors use in effort to assist students, including, but not limited to, direct instruction, error checking, questioning, and hinting (Chi, 1996; Roscoe & Chi, 2008). Examples from these studies show a strong reliance on direct instruction in many cases, which is not surprising considering that most tutors receive little, if any, training, and are not familiar with learning theory (Graesser, Person, & Hu, 2002). When tutors do avoid direct instruction, they tend to use hinting and questioning to guide students toward the tutor's own solution path (Hume, Michael, Rovick, & Evens, 1996; James & Burks, 2018), much like classroom teachers have been shown to use questions in a funneling pattern (Wood, 1998).

This study specifically examines tutoring interactions at a point where the student is needing to focus on the relevant features of a mathematical problem. Students' inattentiveness to particular mathematical features provides a considerable challenge for educators and researchers alike. In multiple areas of mathematics, it has been shown that students can have a solid foundation of specific principles or procedures, yet still be unable to access that knowledge in novel situations (Hoch & Dreyfus, 2005; Schoenfeld, 1980; Selden, Selden, & Mason, 1994). In proof construction, for example, Weber (2001) explains that students who were unable to prove had the necessary syntactic knowledge yet were unable to construct a proof until someone specifically pointed them to the salient facts.

Theoretical Framework

Mason and Spence (1999) refer to this elusive flexibility as knowing-to act. They argue that instruction focuses almost exclusively on knowing-that, knowing-how, and knowing-why, yet success in mathematics relies heavily on students knowing-to act in the moment, which requires an “awareness” or a particular “structure of attention” (p. 138). We use this distinction to identify episodes where tutors are assisting students with knowing-to act and we ask: How do tutors notice and respond to students’ ability to know-to?

Our identification involves assumptions on both the part of the tutor and the researchers. First and foremost, we are selecting episodes where we feel confident assuming the students know-*how* to do the things they may not know know-*to* do without the help of the tutor. For example, in one episode we assume the student is familiar with and has successfully applied the product rule in the past, so the focus of our exploration is his need to recognize the utility of the product rule in his situation, rather than his ability to apply it without error.

In identifying and analyzing episodes, we use a constructivist lens, believing each individual must construct their own meanings (Thompson, 2013). We recognize the tutor and student cannot know what is in the mind of the other. According to Steffe and Thompson (2000), each must create a model of what the other is thinking and react to the other based on that model, rather than what the other is actually thinking. Similarly, we as researchers cannot know what the tutor or student is thinking, so we must formulate models for how we conjecture that both the student and tutor are thinking about the mathematics and their interaction with one another.

We are intentional about viewing constructivism as a learning theory and not a prescriptive teaching method. We believe students can construct meaning for themselves, for example, while listening to a well delivered explanation from a peer tutor. We are not *evaluating* tutors’ responses or methods; rather, we wish to discover what peer tutors believe is helpful to students in a moment of not knowing-to.

Methods

The subjects of this study were eight undergraduate peer tutors at a large public university who work as drop-in tutors for the university’s mathematics department. The tutors had a variety of experience and differing amounts of training. As part of their in-service training protocol, the tutors were required to record a portion of their interactions with students using a Livescribe pen which captured both audio and video of their written work. Each tutor then selected one 5-10 minute episode to transcribe and reflect upon through a written self-evaluation and debrief interview with their supervisor. The interviews between tutors and supervisor were subsequently transcribed by the researchers and pseudonyms were assigned to the tutors.

This study identifies and analyzes tutoring episodes, rather than tutors, because individual tutors often take varying approaches at different times, even within the same short episode (Nardi, Jaworski, & Hegedus, 2005). For this study, we narrow our focus to episodes where the student is asking the tutor for assistance with an issue of knowing-to. To be classified as a knowing-to episode, the tutor must appear to be providing assistance in directing the student’s attention to salient mathematical features of their problem. For example, in one episode, the student must recognize that an expression is two functions multiplied together and decide to use the product rule to differentiate.

We attempt to compare and categorize three episodes by the tutor’s strategies and their intentions as well as the student contributions. While we may describe the tutor strategies and student contributions from the tutoring episode itself, we rely on the tutor’s transcription of the episode, the tutor’s written reflection, and the interview transcript to make conjectures regarding the tutor’s intention for their strategies. We classify the episodes according to the focus or vision of the student and tutor and contrast them based on who was deciding the next move and how the tutor responded to student. We do not claim the three episodes are exhaustive or representative.

Results

Case 1: Divergent Tutor and Student Vision

Case 1 provides evidence that a tutor may not recognize that a student knows-to act if what the student knows-to do is not what the tutor knows to do; that is, if their proposed solution methods differ. In this episode, the student asks for assistance in finding the area of the shaded region, shown below in Figure 1. As the dialogue progresses, we notice that the tutor, Jane, and her student are attending to different aspects of the geometrical figure.

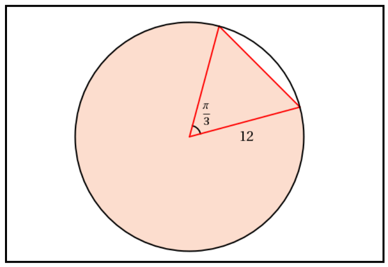

Figure 1: Find the area of the shaded region. (Stewart, Redlin, & Watson, 2016).

Jane: So what do you think are the formulas we're going to be using to go about this are?
Student: We'll use area of a triangle, and then area of whatever this is [area of major sector].
Jane: Yes, so we are on the right track. So the first is area of a triangle. So what's the formula?
Student: ½ ab sine theta.
Jane: Yep, that's correct. Okay, so that will give us the area of this triangle right here, correct? Okay, so we are going to need that. So we are also going to want the area of this [minor] sector because if we get the area of the sector and subtract off the area of the triangle it will give us the area of the non-shaded region. Do you see that?
Student: Yeah
Jane: So it's really area of the sector minus area of the triangle equals area unshaded. And then we have the area of the unshaded we can subtract off the area of the whole region…
Jane: What's our theta?
Student: Um, pi, oh wait, would it be this? [indicating the angle of the major sector]
Jane: …you are right; if we were doing the outer sector then we would use 5 pi over 3, but because we are looking at this sector with the triangle inside it, we're going to do the pi over 3. Does that make sense?

While Jane does not actually ask the student to suggest solution methods, the student's answers to Jane's prompts for formulas and calculations suggest that the student knows-to add the areas of the major sector and triangle. Jane, instead, knows-to find the area of the minor sector and subtract the area of the triangle to get the unshaded region, and then subtract the unshaded region from the circle. Jane does not recognize that the student knows-to because the student's proposed method differs from Jane's. It is not until the interviewer prompted Jane to consider it that Jane acknowledged that the student's path might be viable.

Case 2: Student Vision Unknown

Here we present evidence that tutors may eliminate the opportunity and necessity for students to know-to. In some of these situations, tutors interpret hesitation as not knowing-to. In others, tutors give direction without first giving students an opportunity to demonstrate whether they know-to. In the episode below, the tutor, Abby, asks for student input on the next move, and after a four second pause, explains what to do next.

Abby: So what is the first thing you have to do to take the derivative?
Student: (Ummm…awkward looking at me, indicating they're not sure)
Abby: Okay, so you notice how there's two functions, right? There's x to the fifth and three minus x to the sixth?
Student: Yeah.
Abby: Those are your two, so that means that you use the product rule, right?

During the interview, Abby explains, "…basically they're just like 'umm…'…I interpreted it as 'I don't know where to start on this…I decided to point out, like help them see, like there are actually two functions, like two things multiplied together, and that means that we have to do the product rule."

Later in the same episode, the student is attempting to find the zeros of the derivative. Rather than asking for the student to provide direction this time, Abby provides it herself.

Abby: Awesome. So we have this, and you said earlier that we set it equal to zero. So I'm gonna rewrite it…look good?
Student: Yeah.
Abby: Okay, so it's kind of hard to determine what the zeros are of this function whenever we're, like, adding something in the middle, sooo this is when you want to get things multiplied together.
Student: Okay.
Abby: And you do that by factoring.

Abby explains in her interview that she made this decision to save time, saying "personally, I was like, let's just guide them through this instead of like trying to get them to do it by themselves because that will be quicker." She is not suggesting she knew it would never have occurred to the student to factor, but that it would have taken him longer to realize it than her.

Case 3: Gradually Narrowing Student Vision

At the point where we pick up the episode below, Felicia is helping a student simplify $(1+\cos x)/(\sin x \cos x + \sin x)$. Felicia asks the student to provide direction for the session multiple times. At points where the student appears to be at an impasse, Felicia assists in the knowing-to process by incrementally narrowing the student's focus.

Felicia: …what do you think you're gonna do next?
Student: I kinda was stuck at that point.
Felicia: Okay! So let's look…so we're trying to get some similarity either in the top or the bottom, so that maybe we could cancel something out or just make this guy simpler, right? So, do you see any similarities or any way you could make things look a little bit simpler?
Student: Not really, no.
Felicia: No?
Student: Like the cosine on top and bottom maybe?

Felicia: Um, yeah, there's definitely that. We could try to work with that.... Is there a way you could make that denominator a little bit simpler?

Student: Sine *x* times cosine *x* plus sine *x*…shoot.

Felicia: Well, let's think about it this way. Is there anything that is similar to each of these two terms? (underlines each of the two terms in the denominator)

Student: Both of them have sine.

Felicia: Yeah! So maybe we could?

Student: Take a sine out.

Similar to Abby in the first part of her episode, Felicia first gives her student an opportunity to make the decision for where to go next. However, her response to the student's hesitation is quite different. Felicia seems to be operating under the assumption that the student does know-to. She even challenges the student's response of "Not really, no," when asked if he could make the expression simpler. When the student is stuck or suggests something unexpected, Felicia first suggests some general strategies. ('make it simpler,' 'cancel something out,') When that fails, Felicia narrows the field of vision; she directs the student's attention to *where* he can make it simpler or cancel something out.

Summary

The three episodes illuminate two important variables in tutors' strategies for assisting students with issues of knowing-to. First, tutors decide whether to provide an opportunity for the student to demonstrate whether they know-to. Abby, for example, asked her student how to proceed in the first part of her episode, while in the second part, she stated that factoring was appropriate without asking for input from the student. Although we have some evidence from Abby's episode that time limitations may encourage this strategy, we note that we do not know what motivates these different decisions in different situations and we reiterate that the same tutor can choose different strategies in different scenarios.

The second variable we recognize is how tutors respond to students' indications of their knowing-to or lack thereof. Students can provide solicited or unsolicited information indicating they know-to and tutors can build on or ignore this information. As we see from Jane, tutors may miss a student's knowing-to if it differs from their own. Students can also be silent or verbally claim to not know-to. We see from Abby's episode that a tutor may interpret silence or hesitation as the student not knowing-to. In contrast, as Felicia demonstrated, tutors may not accept a student's claim that they do not know-to. Felicia shows us that tutors are capable of gradually narrowing a student's focus to salient features in a way that still allows them to demonstrate knowing-to at various stages.

Discussion

Future research will provide additional cases and potentially refine those shown above. We seek input regarding the following questions.

- As we analyze more episodes and extend/modify our classifications, should we differentiate based on tutor strategies, tutor intentions, or student responses?
- What existing research, which examines similar phenomena between *instructors* and students, could we build upon in classifying these interactions?
- How do we use this information to design effective tutor training?

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