The Physics Inventory of Quantitative Reasoning: Assessing Student Reasoning About Sign

Alexis Olsho
University of Washington

Suzanne White Brahmia
University of Washington

Andrew Boudreaux
Western Washington University

Trevor Smith
Rowan University

An increase in general quantitative literacy and discipline-specific Physics Quantitative Literacy (PQL) *is a major course goal of most introductory-level physics sequences—yet there exist no instruments to assess how PQL changes with instruction in these types of courses. To address this need, we are developing the* Physics Inventory of Quantitative Literacy (PIQL), *a multiple-choice inventory to assess students' sense-making about arithmetic and algebra concepts that underpin reasoning in introductory physics courses—proportional reasoning, covariational reasoning and reasoning about sign and signed quantities. The PIQL will be used to not only to assess students' PQL at specific points in time, but also to track changes in and development of PQL that can be attributed to instruction. Data from early versions of the PIQL suggest that students experience difficulty reasoning about sign and signed quantities.*

Key words: Signed Numbers, Negative, Quantity, Physics, Reasoning Inventory

(Physics) Quantitative Literacy

Quantitative literacy (QL) plays a major role in everyday life, affecting how one views general risk, and health and economic choices; quantitative literacy facilitates performance on many tasks. Both everyday sense-making and workplace performance rely on QL, and many K-12 and higher education systems have undertaken systematic attempts to improve student performance, yet progress remains elusive (Madison & Steen, 2003; Steen, 2004). We argue that physics, as perhaps the most fundamental and transparently quantitative science discipline, should play a central role in helping students develop quantitative literacy. We coin *Physics Quantitative Literacy (PQL)* to refer to the rich ways that physics experts blend conceptual and procedural mathematics to formulate and apply quantitative models. *Quantification*, a foundation for PQL, is the use of established mathematics to invent novel quantities to describe natural phenomena (Thompson, 2010; Thompson, Carlson, Byerley, & Hatfield, 2014). Quantification is at the heart of experts' investigation of patterns and relationships, which in turn anchor the quantitative models that are the hallmark of physics. Galileo famously wrestled with the mathematical decision of whether to describe accelerated motion with a ratio of change in velocity to distance traveled, or to elapsed time. Choosing the latter led to the formal concept of acceleration, a foundation for Newtonian synthesis.

Quantification relies on blending physics meaning with a conceptualization of the multiplicative and other mathematical structures of the quantities involved; cognitive blending theory helps to frame this blend (Bing & Redish, 2007; Fauconnier & Turner, 2008). Figure 1 illustrates a double scope quantity reasoning blend, in which two distinct domains of thinking are merged to form a new cognitive space optimally suited for productive work. Findings by Czocher support this view. They observed students enrolled in a differential equations course solving a variety of physics problems, and found that successful students functioned most of the time in a "mathematically structured real-world" in which the students moved back and forth fluidly between physics ideas and mathematical concepts (Czocher, 2016). Fluency within this blended space is a hallmark of PQL. We argue that assessing students' PQL gives us insight into the desired cognitive blend.

Though improvement of PQL is a primary course goal, There is little research to assess how PQL develops throughout a typical introductory physics sequence. To address this need, we are developing the Physics Inventory of Quantitative Literacy (PIQL). The PIQL is an assessment instrument intended to probe students' proportional reasoning, covariational reasoning, and reasoning about sign; these three areas are at the heart of quantification in introductory physics (Sherin, 2001; Thompson, 2010; Thompson et al., 2014).

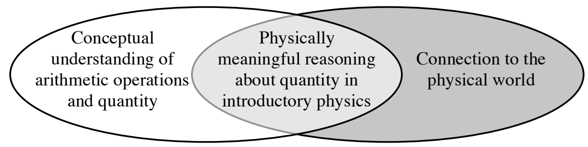

Figure 1: Cognitive blend required for sense-making of physics quantities

In this paper, we discuss recently collected data from a prototypical version of the PIQL (the 'protoPIQL') to preview the types of analyses we hope to achieve using data from more final versions of the PIQL. Our focus in this preliminary report is on instrument items that foreground student reasoning about sign and signed quantities in introductory level physics.

Reasoning About Sign and Signed Quantities

There has been significant research about the different meanings of the negative sign, and student understanding of 'negativity' (Bishop et al., 2014; Vlassis, 2004). Findings indicate that algebraic success is associated with greater 'flexibility' with negativity—that is, students that are able to interpret correctly its use in different contexts show improved performance on tasks such as polynomial reduction (Vlassis, 2004). Flexibility with negativity is analogously important in introductory-level physics, yet no analogous research has been conducted in physics contexts. This paper describes our effort to probe the published natures of negativity (Vlassis, 2004) in a physics context.

Table 1: A map of the different uses of the negative sign in elementary algebra (Vlassis, 2004)

Unary (Struct. signifier)	Symmetrical (Oper. signifier)	Binary (Oper. signifier)
Subtrahend	Taking opposite of, or	Completing
Relative number	inverting the operation	Taking away
Isolated number		Difference between numbers
Formal concept of neg. number		Movement on number line

Table 1, reproduced from Vlassis's 2004 paper, is a map of different algebraic meanings of the negative sign. It served as a guide in our preliminary study of student understanding of the negative sign in introductory-level physics. To begin to probe the effect of introductory-level physics instruction on development of flexibility with negativity, we modified existing signed-quantity questions (Brahmia & Boudreaux, 2017) for use on the protoPIQL. Examples of such questions, and how they fit into the map summarized by Vlassis, are shown in Figure 2.

Figure 2: Items used on protoPIQL representing different algebraic natures of negativity. The acceleration item (left) probes student understanding of the unary (structural signifier, direction of vector component) aspect of negativity, while the work item (center) represents a binary (symmetrical, decrease in system energy) aspect. The position item (right), represents a binary (operational signifier, position relative to origin) nature.

An object moves along a line, represented by the x-direction, and the acceleration is measured to be $a_x = -8$ m/s^2. Consider the following statements about this situation. Select the statement(s) that **must be true.** ***Choose all that apply.***

a. The object's speed is decreasing.
b. The magnitude of the acceleration is decreasing.
c. The object is doing the opposite of accelerating.
d. The acceleration is in the negative x-direction.

A hand exerts a horizontal force on a block as the block moves along a frictionless, horizontal surface. For a particular interval of the motion, the hand does $W = -2.7\ J$ of work on the block. Consider the following statements about this situation. Select the statement(s) that **must be true.** ***Choose all that apply.***

a. The work done by the hand is in the negative direction.
b. The force exerted by the hand is in the negative direction.
c. A component of the force exerted by the hand is in the direction opposite to the block's displacement.
d. Energy was taken away from the hand system.
e. Energy was taken away from the block system.

A person is moving along a line, represented by the x-direction. At a specific instant of time the person is at position $x = -7\ m$. Consider the following statements about this situation. Select the statement(s) that **must be true.** ***Choose all that apply.***

a. The person moves in the negative direction.
b. The person is to the negative direction from the origin.
c. The person is facing backwards.
d. The person is moving backwards.

Methods and Analysis

PIQL is designed as a multiple-choice instrument for collecting quantitative data. Quantitative methods are well-suited to our current investigation, as we are not probing students' 'in-the-moment' thinking. Rather, we hope to track changes to and development of PQL over the course of instruction in introductory physics.

The protoPIQL was administered to $N \sim 1000$ students enrolled in each of the three quarters that constitute one academic year of the calculus-based introductory physics sequence at a large, public American university at the beginning of the academic quarter, before significant instruction had occurred. Therefore, for students enrolled in the first quarter of the sequence, the protoPIQL serves as a pretest for the entire introductory physics sequence. For students enrolled in the second and third quarters of the sequence, the protoPIQL acts as a post-test for the previous quarter's course. Thus we are able to determine whether flexibility with negativity in physics changes over the first two quarters of the introductory sequence. In addition, we wished to investigate how flexibility across contexts is correlated with flexibility within a single context, as described below.

The protoPIQL consisted of 18 questions total: 10 on proportional reasoning, 6 on reasoning about negative quantities, and 2 on covariational reasoning. We focus here on the results of the three negativity questions presented in Figure 2. These three questions represent three different natures of negativity in introductory physics. For a_x, the x-component of acceleration, a negative sign indicates the direction of the vector component relative to a coordinate system. Although the position x is also a vector component (position $\vec{r}$ is a vector quantity), it can be considered an 'almost scalar' quantity in this context, as a one-dimensional position measurement along an axis differs from a location on a number line only in its units. Work W on a system is a scalar quantity that is related to changes in the mechanical energy of a system via the work-energy theorem ($W_{net,ext} = \Delta E$); therefore negative net work on a system indicates that the mechanical energy of that system is decreased. In this case, with only one force that does work on the system, negative net work also indicates that the force and the system's displacement have components in opposite directions, as $W = \vec{F} \cdot \Delta\vec{x}$. Thus, a full understanding of negative work requires flexibility within the single context, as multiple interpretations of the negative sign are possible and (in fact) desired.

Changes in Flexibility With Instruction

For our first, preliminary investigation into changes in flexibility with negativity over the introductory physics sequence, flexibility was defined in terms of answers to these three questions. A small percentage of students did not answer the position question correctly; these students were not given a flexibility designation, as we see understanding of the negative sign associated with position as the most basic understanding of a negative quantity (most analogous to a location on a number line). These students, categorized "Nx" were not included in the following analyses. Students answering only the position question correctly were categorized as "Inflexible" (In). Students that answered only one of the acceleration and work items in addition to answering the position item correctly were categorized as "Intermittently flexible." Students answering all three of the mechanics negativity questions correctly were categorized as "Flexible."

Results for students enrolled in a standard introductory physics sequence (labeled Quarter 1, 2, and 3), as well as students in the third quarter of an 'honors' introductory physics sequence (Q3 Honors) and physics graduate students (G) are shown in Figure 3. Although we see an increase in flexibility after a single quarter of instruction (that is, from Q1 students to Q2 students), there is no significant increase in flexibility thereafter.

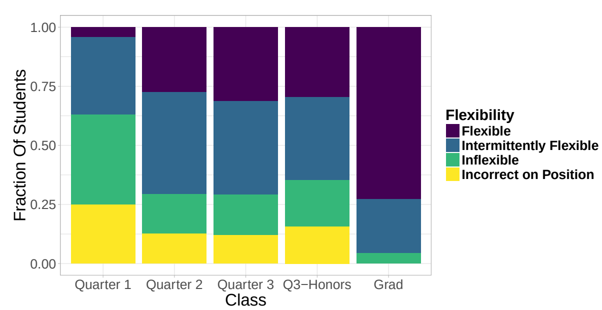

Figure 3: Flexibility for 5 populations of students

Flexibility Within Contexts

To investigate the correlation between flexibility *across contexts* (as above) and flexibility *within a single context*, we consider only students enrolled in the last quarter of the introductory physics sequence (students in Q3 and Q3H, $N = 317$). We also define flexibility differently for this analysis, using a slightly different subset of items: the position and acceleration questions described above, and a third item regarding the meaning of the negative sign associated with a component of an electric field, E_x. Mathematically, the meaning of the negative sign in the electric-field context is similar to that in the acceleration context. We collapse the four categories above into two—students answering zero or one items out of the three were considered to be inflexible, while students answering two or three of these items correctly were considered to flexible. By this metric, approximately 75% of Q3 and Q3H students are flexible (comparable to the sum of Flexible and Intermittently Flexible in Figure 3).

The work item has two correct responses, one that connects the meaning of the negative sign to the relative orientations of the factor vectors $\vec{F}$ and $\Delta\vec{x}$, and one that relates to the system's decrease in mechanical energy. A complete understanding of the negative sign of work requires flexibility within this single context—the negative sign has two correct interpretations. To look at whether inter-context flexibility was associated with intra-context flexibility, we compared performance on the negative work item for students that were rated as inflexible or having emerging flexibility as

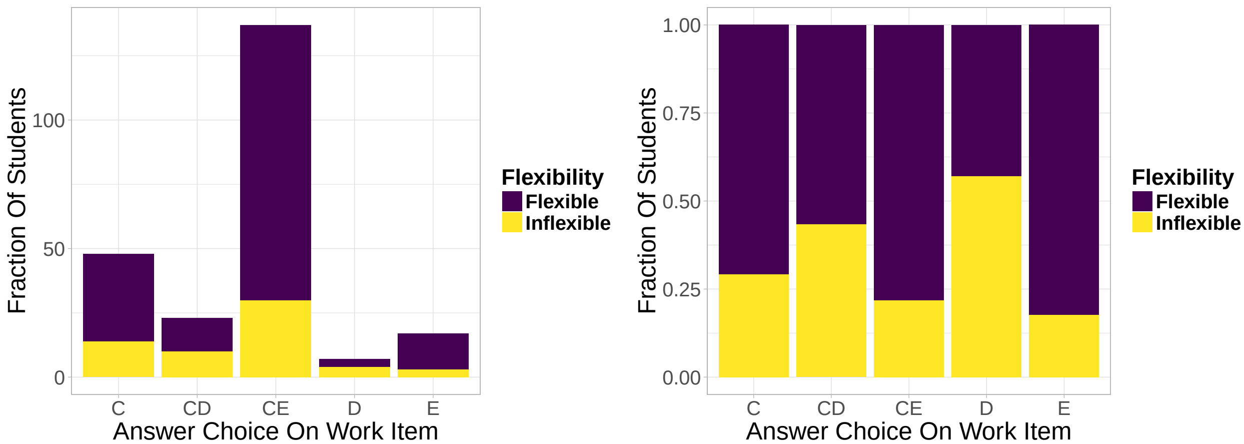

Figure 4: Left: number of students rated as flexible or inflexible based on answer choices. Right: conditional probability of being categorized as flexible or inflexible given answer choice(s).

defined above. (Recall that approximately 75% of these students are flexible by this definition.) The results are shown in Figure 4. Answer choice C compared the relative orientations of the factor vectors of the scalar product, while answer E relates the negative sign to the system's decrease in energy. Answer choice D incorrectly identifies negative work with an *increase* in system energy. X^2 analysis suggests that showing intra-context flexibility by recognizing both possible meanings of the negative sign) is associated with inter-context flexibility ($p = 0.024$). We interpret this result as an indication that flexibility across multiple contexts may help prepare students for the more challenging contexts typical in subsequent physics courses in which there are multiple meanings of signs in a single mathematical statement.

Comments and Future Work

Although the negativity items of the protoPIQL are yielding interesting findings, we believe that our current analysis is limited by the negativity framework developed in the context of algebra. We are developing a negativity framework specific to introductory physics. We find that uncovering the natures of mathematical objects that play multiple roles in physics to be a productive framework for assessment, instruction, and curriculum development. In a related paper in these proceedings, we discuss the Nature of Negativity in Introductory Physics. Such a physics-specific framework will, in turn, necessitate the construction of new items for the PIQL and may inform natures of negativity in the context of quantity used in mathematics education.

Regarding PIQL more broadly, we are creating an analogous map for the natures of covariational reasoning in introductory physics that draws on the extensive work done in the context of mathematics (Carlson, Oehrtman, & Engelke, 2010), and have made progress on a framework for proportional reasoning in the context of physics (Boudreaux, Kanim, & Brahmia, 2015).

This material is based upon work supported by the National Science Foundation under grant number IUSE:EHR #1832836.

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