Theoretical Report: A Framework for Examining Prospective Teachers' Use of Mathematical Knowledge for Teaching in Mathematics Courses

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This theoretical report addresses the challenge and promise of improving prospective secondary mathematics teachers' experiences in undergraduate mathematics courses through tasks embedded in pedagogical contexts. The objective of this approach, used by multiple nationally-funded projects, is to enhance the development of teachers' MKT. We report on the construction of a framework for observing and analyzing the development of teachers' MKT. This framework is the result of integrating several existing frameworks and analyzing a sample of prospective secondary teachers' responses to tasks embedded in pedagogical contexts. We discuss the methods used to build this framework, the strengths and weaknesses of the framework, and the potential of the framework for informing future work in curriculum design and implementation.

Keywords: Mathematical knowledge for teaching, Secondary teacher preparation, Educative curriculum

Recent years have seen multiple nationally-funded efforts to improve the mathematical preparation of teachers by developing materials for undergraduate mathematics courses.[1] Underlying these projects is recognition that mathematics courses are an opportunity to develop mathematical knowledge for teaching (MKT) in ways that are connected to undergraduate mathematics. This opportunity is all too often missed (e.g., Goulding, Hatch, & Rodd, 2003; Ticknor, 2012; Wasserman, Weber, Villanueva, & Mejia-Ramos, 2018; Zazkis & Leikin, 2010).

Scholars have proposed that an important strategy for bridging the gap between mathematical preparation and teaching practice is the use of tasks embedded in pedagogical contexts (Lai & Howell, 2016; Stylianides & Stylianides, 2010; Wasserman et al., 2018). Pedagogical contexts can support teachers' learning of mathematics in ways that are more meaningful and accessible than pure mathematics tasks when it comes to developing MKT (Stylianides & Stylianides, 2010). We conceive of such tasks as approximations of mathematical teaching practice (cf. Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson, 2009), which show promise for helping teachers transfer notions of upper level undergraduate mathematics to secondary mathematics teaching (Wasserman et al., 2018). Wielded skillfully by mathematics faculty teaching university courses, approximations of mathematics teaching practice provide opportunities "absent in fieldwork, [that allow] novices greater freedom to experiment, falter, regroup, and reflect" (p. 2076) when applying mathematical knowledge to the work of teaching.

Problem Addressed

This theoretical report addresses a potential obstacle to enacting approximations of mathematical teaching practice with prospective secondary teachers. Enacting approximations of

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practice has shown promise in small-scale studies involving mathematics education researchers (e.g., Lischka et al., 2017; Wasserman et al., 2016); however, we must take into account contextual differences among preparation programs when scaling up their use. Mathematics education researchers can draw on their field-specific expertise when analyzing teachers' knowledge and providing feedback on teachers' responses to approximations of practice. In contrast, mathematics faculty teach mathematics content courses for teachers in many preparation programs, particularly at the secondary level (e.g., Murray & Star, 2013). Although their experiences may include years of teaching at the undergraduate level, mathematicians' backgrounds are more likely focused on doing mathematics and providing purely mathematical feedback to students. Indeed, even when mathematicians want to use tasks with pedagogical context because they value developing teachers' MKT, they may feel stymied by not knowing how to evaluate prospective teachers' work on such tasks, let alone provide constructive feedback to the teachers (Lai, 2018). The background of mathematics faculty positions them to analyze and observe *mathematics*, but not necessarily MKT.

In sum, instructional improvement efforts face the problem of simultaneously supporting learners (the prospective secondary teachers) and instructors (the mathematicians) in developing MKT at the secondary level. This kind of simultaneous support is the signature characteristic of educative curriculum materials, which have in the past been used as a resource to shift mathematics instruction in sustained, meaningful ways (Davis & Krajcik, 2005).

In this report, we propose a novel integration of existing observational frameworks for examining MKT and its development for the purpose of examining prospective teachers' use of MKT in mathematics courses. We discuss why existing frameworks alone do not suffice for this purpose. Finally, we argue that our proposed integration supports the process of developing educative curriculum materials for undergraduate mathematics courses that feature approximations of mathematical teaching practice. Indeed, we hold that mathematicians and mathematics educators alike can utilize the integrated framework as useful tool when considering how they might provide opportunities for teachers in their courses to develop MKT.

Conceptual Foundations and Proposed Framework

We interlace theory and practice in the improvement work of creating and enacting educative curriculum materials for developing prospective secondary mathematics teachers' MKT in mathematics content courses. This work includes constructing a framework for observing the development of prospective teachers' MKT in their responses to approximations of mathematical teaching practice tasks.

Method

To do this improvement work, we follow a Networked Improvement Community model, with multiple plan-do-study-act cycles (Gomez, Russell, Bryk, LeMahieu, & Merjia, 2016). In this model, the following processes are mutually informing: developing materials, enacting materials, and constructing a framework for observing and analyzing development of MKT. Upon completing three plan-do-study-act cycles focused on observing the development of MKT based on prospective secondary teachers' responses to pedagogically embedded mathematics tasks—specifically approximations of mathematical teaching practice—three principles have emerged to guide our development of a framework. Namely, the framework must: (1) be grounded in theory for how MKT develops; (2) apply to a range of actions that good teaching entails; and (3) be consistent with what is known about observing ways in which MKT is activated in good teaching practice.

Theory for characterizing the development of MKT

Following Ball and Bass (2003), we construe mathematical knowledge for teaching in broad terms—as the knowledge used in recognizing, understanding, and responding to mathematical situations, considerations, and challenges that arise in the course of teaching mathematics. Moreover, we take MKT to include coherent and generative understandings of key ideas that make up the curriculum (Thompson, Carlson, & Silverman, 2007). In alignment with this principle, Silverman and Thompson (2008) used Simon's (2006) idea of key developmental understandings (KDUs) in combination with Piaget's notions of decentering and reflective abstraction to propose a framework for examining how MKT develops. We take Silverman and Thompson's work as a working theory for characterizing the development of MKT.

One principal characteristic of KDUs is that they are "conceptual advances." That is, when a learner (e.g., a prospective teacher or a K-12 student) has a KDU of a mathematical idea, the learner can perceive of and use mathematical relationships to build new understandings in a way that a learner without the KDU cannot. Simon contended that learners acquire KDUs from multiple experiences and reflection. An important implication is that teachers' possession of a KDU, does not ensure that they will create opportunities for students to acquire KDUs (e.g., Silverman, 2004). Indeed, for a teacher do to so, they must not only use or explain personal KDUs, but also envision instructional activities that promote students' learning of KDUs.

Hence, Silverman and Thompson argued developing MKT involves two abstractions, where abstraction aligns with Piaget's notion of reflective abstraction (1977/2001). The first abstraction results in a teacher's personal KDU for a mathematical idea. The second abstraction is on *learners'* thinking and results in multiple models of how learners may understand the idea and how one may come to such an understanding. Silverman and Thompson conceptualize this second abstraction as Piaget's notion of decentering, resulting in: (1) an image of instructional activities and conversations that would produce these understandings and (2) whether these understandings empower students to learn subsequent related ideas, as Table 1 summarizes.

Table 1. Silverman and Thompson's (2008) Characterization of the Development of MKT

Component	Description	
1	Personal KDU:	Teachers have developed a personal KDU for a particular mathematical idea
2	Decentering: Understanding	Teachers have constructed multiple models of student understandings of the idea
3	Student Thinking:	Teachers have an image of *how* a student may come to these understandings
4	Activities:	Teachers can envision instructional activities and conversations that would result in these understandings
5	Potential for Student KDU:	Teachers can analyze how and whether students who have come to think about the mathematical idea in these ways are empowered to learn other, related mathematical ideas

Applying types of knowledge to teaching actions

It follows from Simon's (2006) and Silverman and Thompson's (2008) theory that teachers need to grapple with experiences that promote the abstractions needed to develop MKT. Moreover, instructors of prospective secondary teachers need opportunities to comprehend how teachers understand MKT. For instance, in an example provided by Grossman et al. (2009), prospective teachers responded to two second grade students, coming up with questions to ask the students, reflecting upon the kinds of responses these questions might elicit, and determining extent to which these responses were productive. Through approximations of practice, teachers have the opportunity to engage with student thinking and mathematics to develop MKT.

Although Silverman and Thompson's work describes components of MKT development, it does not elaborate on where to observe these components in teaching practice or in an approximation of practice. To understand where MKT is activated during teaching, we turn to the Knowledge Quartet, which identifies dimensions of teaching in which knowledge is revealed (Rowland, Thwaites, & Jared, 2016). The Knowledge Quartet's purpose resonates with that of Silverman and Thompson's characterization of MKT development, while its focus is complementary. Both acknowledge that instruction should be informed by coherent mathematical knowledge and predictions about learners. Silverman and Thompson focus on mental actions where the Knowledge Quartet identifies visible actions due to teachers' MKT. The four dimensions in the Knowledge Quartet each pair with contributory codes—descriptions of actions that manifest the dimension. The first dimension, (1) Foundation, includes knowledge of mathematics and its nature. The remaining three are contexts in which Foundation knowledge is brought to bear. They are (2) Transformation, the presentation of ideas to learners in the form of illustrations, examples, and explanations; (3) Connection, the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks; and (4) Contingency, the ability to respond to unanticipated events in the work of teaching.

Silverman and Thompson describe the development of MKT, and the Knowledge Quartet describes actions possible due to MKT. When teachers have a personal KDU and engage in the decentering needed to develop MKT, they can design instructional activities to be more responsive to student thinking as well as analyze students' knowledge more acutely. We interpret Foundation to include a teacher's personal KDUs and Transformation, Connection, and Contingency to be actions informed by decentering, understanding students' thinking, and analyzing students' potential KDUs. We summarize the Knowledge Quartet and its relationship to Silverman and Thompson's work in Table 2.

Table 2. Knowledge Quartet in Correspondence with the Development of MKT

Dimension	Example Contributory Codes	Correspondence to MKT Development Components
Foundation	Awareness of purpose; overt display of subject knowledge; use of mathematical terminology	Personal KDU
Transformation	Choice of examples; choice of representation; use of instructional materials; teacher demonstration (to explain a procedure)	Decentering, Potential for Student KDU
Connection	Anticipation of complexity; decisions about sequencing; making connections between procedures; making connections between concepts; recognition of conceptual appropriateness	Decentering, Activities, Potential for Student KDU
Contingency	Responding to students' ideas; use of opportunities; deviation from agenda	Decentering, Potential for Student KDU

Activation of MKT in teaching practice

Although Silverman and Thompson provide a theory for how MKT develops and the Knowledge Quartet provides a description of how different types of knowledge are applied to aspects of teaching, neither elaborate how one instance of an application of MKT to teaching practice may be more sophisticated than another instance. For this, Ader and Carlson's (2018) work provides a mechanism for distinguishing levels of the sophistication of activation of MKT by describing patterns in observable behaviors during teaching practice that indicate the extent to which the teacher has decentered. We describe our view of the correspondence of these levels and observable behaviors to Silverman and Thompson's characterization of MKT in Table 3.

Table 3. Ader and Carlson's Framework in Correspondence with MKT Development

Level	Observable behaviors	Correspondence to MKT Development Components
Level 1: Interested in getting students to say correct answers but not in students' thinking	Asks questions to elicit students' answers; listens to students' answers; does not pose questions aimed at understanding students' thinking	*Decentering*: lack of decentering, uses only a first order model; *Understanding Student Thinking*: only elicits student answers, not thinking; *Activities*: constrained by thinking only with first order model
Level 2: Interested in students' thinking, but only in order to get students to think like the teacher	Poses questions to reveal student thinking but does not attempt to understand students' thinking; guides students toward his/her own way of thinking.	*Decentering*: lack of decentering, uses only a first order model; *Understanding Student Thinking*: only elicits student thinking, does not utilize that thinking in a response; *Activities*: constrained by only thinking with a first order model
Level 3: Makes sense of students' thinking and makes general teaching moves based on that thinking	Asks questions to reveal and understand students' thinking; follows up on students' responses in order to perturb students in a way that extends their current ways of thinking; attempts to move students to his/her thinking or perspective	*Decentering*: evidence of first and second order models; *Understanding Student Thinking*: utilizes student thinking when formulating responses; *Activities*: uses second order model to make decisions about activities and conversations; *Personal KDU*: draws on personal KDU when responding to students
Level 4: Seeks to understand students' thinking, and builds on that thinking during instruction	Poses questions to gain insights into students' thinking; draws on students' ways of thinking to advance students' understanding of key ideas in the lesson	*Decentering*: evidence of first and second order models; *Understanding Student Thinking*: utilizes student thinking when formulating responses; *Activities*: uses second order model to make decisions about activities and conversations; *Personal KDU*: draws on personal KDU when responding to students; *Potential for Student KDU*: seeks to provide opportunities for students to develop KDUs

Working Framework for Observing and Analyzing the Development of MKT in Approximations of Mathematics Teaching Practice Used in Content Courses

In Networked Improvement Community work involving multiple rounds of coding prospective secondary teachers' responses to approximations of mathematical teaching practice, we began by using the dimensions and components in Tables 1 and 2. We found it difficult to determine the *development* of prospective secondary teachers' MKT over time. To remedy this issue, we incorporated and generated hypothesized extensions of the correspondence of levels (Table 3) for each dimension (Table 2).

Method for extending levels. To construct extensions of levels, the six authors analyzed the responses of 15 prospective secondary teachers to approximations of mathematical teaching practice that were has been used in mathematics courses at 3 different institutions in different states in multiple years; the responses analyzed were representative of the responses across these sites. We adapted a two-stage coding process (Miles, Huberman, & Saldana, 2013), using the dimensions of the Knowledge Quartet as *descriptive codes* and Ader and Carlson's levels as initial *process codes* in a first cycle of coding, and then used a second cycle of coding to consolidate codes for structure and unity. To do so, we drew on critiques of episodes of teaching found on the Knowledge Quartet's website (Rowland, 2017) and observation protocols that have been validated as measuring quality of teaching (Junker et al., 2004; Learning Mathematics for Teaching, 2011).

Results. We interpret our work has contributing several results. Our first result is theoretical: the framework to which our analysis led. This framework is presented in Table 4.

Second, as a practical result, we report which aspects of the framework led to the most and least reliably coded approximations of mathematical teaching practice.

We describe this second result in brief here, for the sake of space limitations, and provide more elaboration in our presentation. The first-cycle descriptive codes for the dimensions of the Knowledge Quartet, as well as the process codes for the levels for Transformation, were most reliably coded among the research team. The least reliable codes were Levels 2 and 3 within Connection, as well as the Level 4 codes for Transformation and Connection. We see reliability of codes as an important result to report because it bears on interpreting the framework as well as pointing to future work in validating this framework for observing the development of MKT.

Table 4. Framework for Observing and Analyzing the Development of MKT in Approximations of Practice

Developmental component	Knowledge dimension	Mental actions	Level (L), in terms of observable behaviors
Personal KDU	Foundation	Reflective abstraction on personal mathematical knowledge	*Note: Levels here depend on the KDU of the topic. This is just one possible example of how levels may appear.* L0: Specific reference to mathematics is not present OR Performs procedures incorrectly and describes underlying concepts incorrectly (lacking in CCK) L1: Performs relevant procedures correctly L2: Describes relevant procedures accurately, with mathematically precise and appropriate language L3: Connects isolated features of procedures to underlying concepts L4: Connects structure of procedure to underlying concepts
Decentering applied to Activities and Analyzing Potential for Student KDU	Transformation	Reflective abstraction on student thinking	Gives explanations, representations, and examples to students that: L0: Does not provide any explanations, representations, or examples to students L1: Describe only procedures or echo key phrases L2: Describe own way of thinking of the mathematics L3: Attempt to change students' current thinking L4: Build on and respect students' understanding toward the intended KDU
	Connection		Prompts students to say or do things in ways that: L0: Does not ask students to say or do anything L1: Focus on procedures or echoing key phrases L2: May reveal student thinking, but then teacher gives explanations while not asking students to provide reasoning L3: Attempts to change students' current thinking L4: Build on and respect students' understanding toward the intended KDU
	Contingency		Uses student thinking in ways that: L0: Do not act in any visible way upon the thinking L1: Evaluate the mathematical validity of the thinking but do not use the thinking in teaching L2: Reference the thinking to guide students toward teacher's way of thinking L3: Follow up on students' responses to perturb students to change their thinking L4: Frame questions or explanations in terms of students' thinking to help move students' understanding toward the intended KDU. Students are positioned as decision-makers.

Discussion

Our work is built from conceptual foundations to elaborate how and where teachers' development of MKT can be observed and analyzed, both by all those who teach teachers as well as by mathematics education researchers. Silverman and Thompson's framework provides a characterization of how MKT develops, in terms of mental actions of teachers, leaving open where in teaching to observe these mental actions and what observable behaviors those mental actions might produce. The Knowledge Quartet describes where the results of teachers' mental actions show up in teaching practice, and Ader and Carlson's framework characterizes the relative sophistication of those mental actions in terms of observable behaviors. Approximations of mathematical teaching practice engage prospective teachers in teaching actions and provide opportunities for teachers to engage in the mental actions needed to develop MKT.

The framework we present supports mathematics faculty and teacher education researchers in discerning knowledge use in approximations of practice. The dimensions of Foundation, Connection, and Transformation emphasize places where prospective teacher might display personal knowledge, provide explanations to students, and pose questions that elicit student reasoning. Although faculty may not traditionally provide feedback on these distinctions in a mathematics course, these distinctions are ones that may be familiar to faculty and may well be educative for their own teaching practice (e.g., Bass, 2015; Pascoe & Stockero, 2017). Our data suggest that the dimensions of knowledge were independent, providing evidence that pathways through development of MKT may well proceed along these dimensions in different ways. For instance, one prospective teacher in our dataset explained the connection between a definition and procedure as a rationale for a task they would assign to their students (Foundation, L4), but only posed questions that focused on echoing key phrases (Connection, L1), proposed only explanations of procedures to the students (Transformation, L1), and did not acknowledge any of the sample student thinking provided by approximation of practice (Contingency, L1). Another teacher began with using the provided sample student work to make a specific mathematical point about a definition (Contingency, L4) then did not provide any subsequent examples or explanations to connection of procedure and definition (Transformation, L1).

Upon receiving initial feedback from mathematics faculty regarding how observables may correspond to knowledge dimensions, our work writing approximations of mathematics teaching practice for use in content courses shifted to address more clearly the specific opportunities for learning that those approximations provide. For instance, in an early draft of an approximation of practice, we asked teachers to respond to student thinking, but we did not give a clear mathematical goal for the teaching situation. This left unspecified the Foundation knowledge we were aiming to elicit, which impacted the Transformation and Connection knowledge visible in teachers' responses.

Finally, the framework supports validating and refining the articulation of the development of MKT. We view this framework as a set of testable hypotheses grounded in known results. Given the relative nascence of research on developing MKT (Hoover, Mosvold, Ball, & Lai, 2016), such hypotheses can contribute to advancing understanding of MKT.

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