The Impact of an Upper Division Inquiry-Oriented Content Course on Prospective Teachers: Embracing a Critical Stance

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In this study, we analyze interviews with prospective secondary school teachers focused on how an upper division inquiry-oriented content course influenced their beliefs about learning and teaching mathematics. In particular, we examine how students describe their experiences in this course and the extent to which (and why) students see themselves using an inquiry-oriented instructional approach in their future teaching. Using thematic analysis, we found that the ways prospective teachers described their experiences in the course aligned closely with the four pillars of Inquiry-Based Mathematics Education, and the course empowered students to embrace a critical stance to self-reflect on their experiences and to raise their awareness of their agency to change things (for the better) through their own actions and future forms of instruction.

Keywords: critical stance, inquiry, prospective teachers, beliefs about teaching

The relevance and usefulness of upper division mathematics courses for prospective secondary school mathematics teachers has long been of concern (Begle, 1972; Klein, 1932; 2016; Wasserman et al., 2019). A number of studies document that teachers find their advanced mathematics courses have little relevance to their future professional life (e.g., Cofer, 2015; Wasserman, 2017, Zazkis & Leikin, 2010). One approach to addressing this issue, and the one taken in our larger study, is to redesign upper division math content courses to make intentionally strong connections to high school mathematics content and teaching.

A recent issue of *ZDM Mathematics Education* has made progress on the discontinuity (Klein, 1932) between university and secondary school by examining how the intersectional nature of mathematics educational content might be addressed in a wide range of university courses in order to prepare better secondary mathematics teachers (Wasserman et al., 2023). For example, Buchbinder and McCrone (2023) focused on bridging the disciplinary practice of proving and the teaching of secondary mathematics. In another paper in this special issue, one that is related to our larger project, Apkarian and colleagues (2023) investigated the impact of an upper division mathematics course on prospective teachers' knowledge of rate of change and their shifting beliefs about mathematics learning and teaching. In other related prior work that investigated the impact of this same course, Goodchild and colleagues (2021) took a community of practice perspective to document the adoption by these prospective teachers of a *critical stance* toward learning and teaching mathematics. In this report, we further this line of inquiry by examining prospective secondary school teacher experiences in an inquiry-oriented upper division math course and the extent to which they see themselves enacting an inquiry-oriented instructional approach in their future teaching. In particular, we address the following two research questions: *(1) How do students describe their experiences in an upper division math course and how are these experiences similar to or different from their previous college and high school math experiences?* and *(2) To what extent do students see their future selves using an*

inquiry-oriented instructional approach and what are their reasons for doing so (or not doing so)?

The work reported here is part of a larger NSF-funded project that is investigating the impact of an upper division mathematics course, Inquiry-Oriented Dynamical Systems and Modeling (IODSM), on prospective teachers' knowledge of content connected to high school mathematics, the ways in which they engage in the eight Standards for Mathematical Practice (Common Core State Standards Initiative, 2010), their beliefs about learning and teaching mathematics, and contributions to their emerging professional practices.

Theoretical Framework

As alluded to in the introduction, two central and related ideas to our line of investigation are inquiry-oriented instruction and critical stance. The upper division math course that we are redesigning for prospective teachers embraces and reflects the four pillars of inquiry-based mathematics education (IBME): (1) Students engage deeply with coherent and meaningful mathematical tasks, (2) Students collaboratively process mathematical ideas, (3) Instructors inquire into student thinking, and (4) Instructors foster equity in their design and facilitation choices (Laursen & Rasmussen, 2019). Our creation of coherent and meaningful tasks where students reinvent mathematics is guided by the instructional design theory of Realistic Mathematics Education (Freudenthal, 1991; Gravemeijer, 1999) and our perspective on the ways in which students and instructors inquire into others' thinking and reasoning is informed by the constructs of social and sociomathematical norms (Yackel & Cobb, 1996). In particular, the instructor sought to create a classroom environment that nurtured the following social norms: students routinely explain their thinking, however tentative, students listen to and try to make sense of others' reasoning, and indicate agreement or disagreement with reasons. In terms of sociomathematical norms, the instructor sought to foster student explanations grounded in mathematical relationships and meaning, as opposed to procedural accounts of their solution.

For his part, the instructor sought to foster the two student pillars by valuing and bringing out student ideas and then leveraging those ideas to move forward the mathematical agenda (Kuster et al., 2018; Rasmussen & Marrongelle, 2006). The instructor's attention to equity included specific teaching choices such as attending to group composition, explicitly crediting ideas to student authors to foster intellectual ownership, and regularly collecting and leveraging student reflections on each class session. Another choice made by the instructor was to assign rotating group roles. For example, one of the assigned roles was a Reporter who reports back to the whole class about their group's progress which contributes to the instructor's efforts to ensure equitable participation and minimize biases when asking groups to report out (Reinholz, 2023).

The four pillars of inquiry-based mathematics education are foundational for students to develop a *critical stance*. Our definition of critical stance draws on that put forth by Curzon-Hobson (2003, p. 201): A *critical stance* is "an attitude or disposition towards oneself, others and the object of inquiry that challenges and impels learners to reflect, understand and act in the milieu of potentiality." A critical stance therefore results in an awareness of and reflection on one's own experience, meanings, and knowing. In prior related work, Goodchild and colleagues (2021) distilled this characterization of critical stance into three key features: First, students must be aware of the possibility that their mathematical learning experience could be different, for this they need to experience variety in the way mathematical learning experiences are designed and implemented by the instructor. Second, students must be able to reach some judgment, based on their reflection and evaluation of their experience so that they might recognize when one experience of meaning, learning or teaching is better, deeper, or richer than another. Third,

students need to be aware of their agency to change things (for the better) through their own actions and future instructional practice as a secondary school teacher.

Methods

This report focuses on data collected during the Spring 2023 semester. The goal of this pilot study was to begin modifying an existing differential equations curriculum (Rasmussen et al., 2018) so that it explicitly deals with high school mathematics content and teaching and to investigate the impact of this course on prospective teachers' knowledge, beliefs, and their participation in the Standards for Mathematical Practices (Cruz et al., 2024).

We conducted semi-structured individual interviews with nine students from a class of 30 at a large, Hispanic-serving institution in the southwestern United States. Of the nine volunteers, five participants identified as women and four identified as men. One of the authors was the instructor of the class and they did not know who participated in the interviews until after the interviews and course were completed. In this report, we focus on interview questions related to their previous mathematics experiences and beliefs about learning and teaching mathematics as it relates to having a critical stance and their future professional practice. The relevant interview questions were:

- Have you had other math courses taught in a way similar to this class?
- Do you imagine yourself using this kind of teaching in your future practice? If so, how? If not, why not?
- In what ways, if any, have your experiences in this class affected your beliefs about teaching mathematics?
- Are there ways of experiencing or doing math in this class that have been powerful for your learning?
- How do these ways of experiencing or doing mathematics compare to your other undergraduate math courses? To your high school math courses?
- How about ways in which these ways of doing math in this class have affected your beliefs about what it means to learn mathematics?

The interviews took place in person with one of two interviewers participating over Zoom. Interviews were transcribed and anonymized. All names we include here are pseudonyms.

We first read the transcripts to familiarize ourselves with the data. We then used thematic analysis (Braun & Clarke, 2006) to identify, analyze, and interpret patterns within the data. During the initial coding process, we open coded the data with some a priori attention toward the components of a critical stance (awareness, self-evaluation, and agency). We coded in pairs and went through several rounds of reading and updating our codes until a 100% agreement was reached. During axial coding, we grouped codes according to the following five themes: (1) students' awareness of a different experience, (2) intellectual responsibility lies with students, (3) the role of social interactions, (4) instructor actions and decisions, and (5) course and facilitation design choices with an eye towards equity. In further axial coding, we related these themes to our theoretical perspectives of inquiry and critical stance.

Results

Research Question 1. Recall that our first research question is, *How do students describe their experiences in an upper division math course and how are these experiences similar to or different from their previous college and high school math experiences?* Our analysis of the interview data revealed that the pedagogical approaches experienced by students in the IODSM class are very different from their collective prior mathematics experiences. Seven of the nine

students indicated that their previous mathematics, high school or lower division mathematics undergraduate classes, were all or mostly lecture-based where they engaged in "mindless copying", "repetitive practice" and it was all very "fast paced" to keep up with the instructors. The newness of a different mathematical learning experience is addressed in the first of our five themes.

Theme 1: Students' awareness of a different experience. Recall one component of a critical stance is awareness: students must be aware of the possibility that their experience of the practice could be different. Three students in the IODSM course indicated that they were unaware that math could even be taught like this. For example, Chepi said:

> I never thought that a math class could be like this where you're just figuring it out for yourself and trying to make sense of it yourself with other people. You're getting new perspectives rather than your own, because sometimes you could think of things one way. There's no right way to do math, as we've seen and in a class like this, I've seen people do some weird things that you would never think for yourself and it works.

The remaining four themes further flesh out the various ways in which IBME offers a different, and, from students' perspective, more meaningful learning experience.

Theme 2: Intellectual responsibility lies with students. For the second theme students indicated overwhelmingly that they could develop mathematical ideas for themselves, with minimal input from the instructor. For example, George said, "I think that [the] process of having the students really dig into the material with minimal assistance from the instructor is very good." Students also reported feeling confident in reinventing mathematics where they have ownership of ideas. Here, the way this manifested for students was the importance of having intellectual responsibility for their own learning. For example, Hakeem said:

> Because, you're creating something on your own and obviously you're gonna take pride in that. When you have something that you created, you're very proud of it and you're very defensive of it also. So if someone says that you're wrong, you're definitely gonna want to counter argue that and defend yourself. And then they're gonna also, and it's that back and forth, that dialogue is where new ideas and true learning happens.

This theme is strongly connected to IBME Pillar 1, Students engage deeply with coherent and meaningful tasks, because of the intellectual ownership and need to "really dig into the material."

Theme 3: The role of social interactions. Another theme that emerged was the role of social interactions and the power of collaborative learning. The statements coded in this grouping were attributed to eight of the nine students. This theme resonates with the second IBME pillar, Students collaboratively process mathematical ideas, because students positively reflected on the importance of group work, group roles, and the opportunities for student-to-student discussion. For example, the collaborative processing of ideas is evident in Hakeem's reflection when he said "it's that back and forth, that dialogue is where new ideas and true learning happens." Hakeem also emphasized the value and benefit of students being the primary source of ideas:

> I think it's better to have those small groups and be able to talk to them about your understanding of where you're struggling with. I think that's way more beneficial and it just brings more comfort to the student with the material. And then eventually, I'm sure you could see it too, as the semester progresses, more students are being able to talk or come out on their own and being able to share their ideas. Whereas at the beginning of

> the semester, there's only probably like three people that would consistently be asking questions, consistently be sharing. So I definitely think having inquiry-based small group discussions where the lecturer is only there as a resource, not necessarily the source, is the best way to learn.

Theme 4: Instructor actions and decisions. Five students' descriptions of the IODSM course highlighted the instructor choices about the course design that indicated purposeful instructor inquiry into student thinking. For example, Jackie said the following about the end-of-class reflections: "...I think [it] is really nice that we get to see other students thinking in their writing as opposed to just presenting in class", whereas Hakeem commented on how he used the reflections: "I went back on my reflections and on the entire class's too, because [the instructor] posts them… gathering from their knowledge, adding it to my own and making it my own." We see this theme as implicitly aligning with the third IBME pillar, Instructors inquire into student thinking, because the instructor regularly used student thinking from the end of class student reflections to jump start the next lesson (i.e., move forward the mathematical agenda).

Theme 5: Course and facilitation design choices with an eye towards equity. Four of the nine students spontaneously commented that their new experiences in the IODSM course contributed to equitable learning experiences. It is notable that the interview questions did not directly ask about equity and hence student reflections related to this theme emerged organically from the interview data. For instance, Francis realized the facilitation effort that the instructor made to promote group diversity during collaborative working time:

> …I like how, as a student, you get to experience it. So I would want to give my students the same experience just because, like giving diversity, how he [the instructor] puts boys and girls in groups… So it's not just a boys club or a girls club. It's more of everyone just collaborating, working together, getting to know each other… It's not just like, oh, I don't know that person. That's more like, oh hey, how's it going?

Similarly, Jackie observed the impact of the course design in which each student in the group work has a role where they said, "We're in groups and each group member has a different role to help, I guess, function and make sure that everyone has a voice in their group." Gabby also addressed the value of group work when she said, "My favorite part of the class is working together as a class, where like we'll have our time to do our own group work and either some groups are excelling, some are struggling and everyone's kind of trying to keep each other in check." This theme aligns with IBME Pillar 4, Instructors foster equity in their design and facilitation choices, as the group work and group roles allowed for more equitable participation.

Research Question 2. We next address our second research question, *To what extent do students see their future selves using an inquiry-oriented instructional approach and what are their reasons for doing so (or not doing so)?* All nine students answered positively that they would like to use the inquiry-oriented instructional approach in their future practice. For example, Diem said "I like it… I would like to do the same format to teach in the future." Breanne similarly commented "I would want to… And the teacher that I've been shadowing in a high school, she kind of does similar things... everybody sits in little groups and like group presentations… but a little bit dialed down."

Three of the nine students saw themselves implementing similar instructional approaches, but with caveats that the "high school curriculum is packed" and "may not fit the school model." We view these awarenesses of the potential secondary school context as an indication of their critical stance, since they evaluated the practicability of the inquiry-oriented approach in a high school

setting. This shows their agency, transitioning beyond a learner perspective to a teacher perspective in their future professional life. For example, Gabby envisioned that:

> I would really like to. I know it takes a lot more time to do, and that's one of the bigger issues of doing this kind of thing in the classroom and just like the rest of the school not being set up this way or not being supported to set up this way. But ideally, yes. Even if I'm not able to fully do it, I have already started thinking about how I wanna do little things that we do in this classroom and incorporate that into a more traditional [classroom]…

Note that Gabby was explicitly aware that she might take a position in high school where "the rest of the school not being set up this way or not being supported to set up this way." She also started to brainstorm what she should change to integrate inquiry-oriented learning into her future teaching. Similarly, George embraced a critical stance by anticipating similar challenges. In particular, in the following excerpt, he recognized the trade off between covering a lot of material and the depth of understanding:

> But you have to think about it too in the sense that if you do try to fit in more material, it's less likely that the students will have as good of an understanding on the material that they can cover. So you kind of have to weigh how well these students really need to understand a concept versus how much material do they need to know.

These examples highlight how prospective teachers exercised their agency by hypothesizing the balance of time that an inquiry-oriented approach might require in their future teaching.

For the majority of students who see their future selves using an inquiry-oriented instructional approach to a great extent, we observed similar themes in their reasonings aligned with the pillars of IBME. For example, Chepi recognized the benefits and appreciated the IODSM course's collaborative thinking activities to "help kids learn to think for themselves rather than to be taught cookbook how to do anything in math" (Pillars 1 and 2). Many students, like Chepi, reflected on and evaluated their IODSM experiences to reason why they want to use this teaching from the perspective of learners. For example, George said,

> I think that the typical lecture while students just kind of watch model of teaching is outdated and not very beneficial to the students. So this class is just so reinforcing and I think it's very helpful to the students to acquire these ideas on our own and like really work our way through the concepts.

The students again embraced the agency to make the judgmental call that the intellectual responsibility of understanding and sense-making mathematics should lie within learners instead of teachers telling students what to do and how to think. This finding re-emphasizes the themes of students' awareness (Theme 1) concerning students' intellectual responsibility for learning (Theme 2).

Another reason why students favored inquiry teaching and learning is that the experience is "more class engaged, student engaged, rather than just everybody listening to one person talking and writing on the board" (Breanne). Christiano also elaborated on the engaging aspect stating that "it encourages my desire to teach. It makes me excited, because there's definitely other math classes that I've been in, and I definitely see, I think to myself, I don't want to teach like this and in this class… it kind of solidifies my desire." Breanne commented that learning from classmates with group work and the whole class was a "productive struggle" which helped her "retain

information better." In addition, students were well aware and highlighted the collaborative, safe, open, and supportive learning environment— "everyone kind of knows each other, so it's kinda open. No one's too shy to speak up" (Francis) and "I think that's way more beneficial and it just brings more comfort to the student with the material. And…. as the semester progresses, more students are being able to talk or come out on their own and being able to share their ideas" (Hakeem).

Discussion and Conclusion

Overall, results indicate that students' prior mathematics experiences were more different than similar to the pedagogical approach taken in the IODSM course. It is noteworthy that the pedagogical approach they experienced provided opportunities for them to increase their awareness that teaching and learning can be different, in positive ways, from the majority of their prior experience. Moreover, how the students embraced aspects of the pedagogical approach all align with the pillars of IBME. It should be noted that the pillars of IBME were in the mind of the instructor when designing the course structure, but these pillars were not explicitly discussed with students or brought to the attention of the students. Thus, their collective responses in the interviews about their intellectual responsibility, their collaboration with their peers, and their instructors' role in their learning and his attention to equity, all indicate that the IBME pillars were realized in this course.

In this report, we provided evidence that students embraced a critical stance—they were aware of the new mathematical experience with the IODSM course as compared to their previous experiences, were inspired to evaluate the experiences as a learner and a teacher, and recognized their agency to make changes for the better in their future teaching. Drawing from their experiences with the IBME-inspired instructional approach in the IODSM course, students reasoned positively about why they would see their future selves implement an inquiry-oriented instructional approach and the value that such an approach has for learners. Even if they had caveats, students explained their concerns and further proposed how they would modify the inquiry-oriented approach they experienced to adapt to their imaged secondary school settings. Further, student reflections on their experiences in this enactment of an inquiry-oriented course effectively empowered and fostered their embracement of a critical stance. Students were aware of the new mathematical experience, reflected on and evaluated whether the IBME experiences were more profound, and used their agency to imagine enacting an inquiry-oriented instructional approach in their future teaching. Our results provide an existence proof that, in this case, the particular enactment of IBME supported and nurtured a critical stance.

Going forward, in the context of the larger NSF study, we will go beyond interview data and collect classroom video data from other institutions with different instructors and contexts so that we can explore the potential for other IODSM class structures and specific teacher moves and decisions to positively affect future teachers' beliefs about mathematics learning and teaching and their embracement of a critical stance. The larger study also aims to understand how prospective teachers' knowledge of content related to high school mathematics deepens, the ways in which they engage in the eight Standards for Mathematical Practice (Common Core State Standards Initiative, 2010), and how approximations to their professional practices emerge and grow (Llinares, 2013).

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