Exploring Students' Understanding of a Limit of a Sequence: Using ϵ-Strip Activity With Realistic Mathematics Education Framework

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The concept of limit is crucial in undergraduate mathematics because it serves as a foundation to help students make sense of formalized, rigorous mathematics statements as well as its application in calculus, real analysis, and upper-year division mathematics courses (Roh, 2008; Roh & Lee, 2017; Swinyard & Larsen, 2012). However, many studies show that students find developing a robust conceptual understanding of limit challenging (Oehrtman et al., 2014; Swinyard & Larsen, 2012; Williams, 1991).

I interviewed two students to examine if the ϵ-strip activity developed by Roh (2008) could scaffold and help students leverage their intuitive understanding of the limit in a sequence to the mathematically rigorous ϵ-N definition of convergence of a sequence. In particular, I investigated how students reason about the convergence or divergence of a sequence during the ϵ-strip activity. As a theory-building effort, I analyzed the interview data by mapping it into a table figure that integrates the constructs from the Defining as a Mathematical Activity (DMA) framework and Realistic Mathematics Education (RME).

Freudenthal (1991) proposed the RME as an instructional design that aims to foster the invention of mathematical knowledge called "emergent models." It highlights the shift between how students develop *models-of* their informal mathematical activity using tools, graphs, and analytic expressions to *models-for* increasingly sophisticated and formal ways of mathematical reasoning in four phases of Situational, Referential, General, and Formal activities (Gravemeijer, 1999). Zandieh and Rasmussen (2010) further described these four phases with the DMA by integrating the notion of concept definition and concept image (Tall & Vinner, 1981). This integration adds five constructs to the four phases of RME's transition process, which are creating a concept definition, using a concept definition, creating a concept image, using a concept image, and creating a new mathematical reality.

By mapping out the interview data and identifying critical events as explicit evidence for the transition process in a figure, I observed that the ϵ-strip activity aligns well with RME's emergent model and DMA. At the beginning of the ϵ-strip activity, students created a concept image and concept definition of ϵ-strip recalling their past understanding of limit as a model-of, which is what the Situational and Referential activities in an experiential real setting attribute. As they completed more activities, students used the physical ϵ-strip as a model-for to create a concept image and concept definition of the rigorous ϵ-N definition of convergent sequences. In particular, Henry (pseudonym) almost reinvented the ϵ-N definition with De Morgan's Laws by formalizing their concept image, an occurrence of the Formal activity in the wild.

As a result, this study examined the practicability and robustness ϵ-strip learning activity grounded with DMA and RME using a figure to present how the interview data merged into the four levels of activity and five constructs of DMA. Furthermore, I envision that mathematics education researchers and practitioners not only can use the DMA framework to guide the design process of instructional sequences that scaffold complicated mathematical topics related to defining, but they can also use DMA to improve the design of existing instructional sequences like the ϵ-strip activity that has the potential to leverage students' emergent model to formalize their understanding towards rigorous mathematical statements.

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